

Rethinking the history of mathematical symbolism
A cooperative project

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SPHERE

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Second Year: Diagrammatization of written mathematical practices

Organized with Zehra Bilgin, Alex Garnick, Agathe Keller and Xiaohan Zhou

General histories of mathematics commonly take up the view that mathematical symbolism was an innovation introduced in Europe at the end of the sixteenth century. This project aims to explore the hypothesis that what was practiced at the time was rather, more precisely, a new type of mathematical symbolism drawing on previous ones. In other words, the project will suggest rethinking what we mean by 'symbolism'. In particular, we will pursue the contention that mathematical symbolism has, in fact, a much more global history than has so far been assumed.

In its first year, the project began with a historical exploration of the historiographies of mathematical symbolism through a series of seminars and a conference (<https://edin.ac/4aqArkm>). Our aim was to explore the historical shaping of the view that mathematical symbolism originated with sixteenth- and seventeenth-century European actors. The historical importance assigned to their work in this respect has typically derived from their use of literal computation, that is, computations on letters with unspecified values rather than particular numerals. We wanted, in particular, to highlight the facets of symbolism that have been neglected, or that have been treated as mere consequents of other symbolic features perceived to be primary. Our aim was also to examine properties and virtues of mathematical symbolism that some actors have foregrounded in their historical analysis and that may constitute different starting points for a history of mathematical symbolism.

This reflection has brought to light that several historians (e.g., Nesselmann 1842, Neugebauer 1932-1933, Burnett 2002) had emphasized a key feature of symbolism that might serve as a promising starting point for the project, namely, that mathematical symbolisms were a primarily written practice and thus independent from spoken language. This viewpoint has led both Neugebauer and Burnett to broaden the range of notations to be included in a history of mathematical symbolism, albeit each in completely different ways.

The second year will explore this property of symbolisms—their independence from spoken language—by focusing on the diagrammatic dimensions of various mathematical notations and inscriptions, along with their corresponding practices. Indeed, symbolisms often involve non-linear notations, and working with them has required the creation of specific mathematical practices to navigate through them.

Why do we mention both notations and inscriptions? In the ancient and medieval sources, notations of these kinds often appear in margins or are even simply described in

texts, the notations being themselves used only on ephemeral supports—these are the notations for which we use rather the term “inscription”. In all these cases, what is the status and meaning of these notations and/or inscriptions with diagrammatic features and of the practices with them? Are they schematic ways of prescribing or are they diagrams on the basis of which to carry out computation or reasoning? Who drew them and for which purpose? These questions must be addressed, notably because some of the notations that will later feature within the works themselves present a strong continuity with these ephemeral and marginal notations and/or inscriptions (Chemla 2014). In any event, we will consider the assumption that practices that diagrammatize prescriptions, computations or reasonings played a part in the history of symbolic work in mathematics.

Such a focus on the diagrammatic dimensions of mathematical notations and/or inscriptions as well as of practices with them appears promising for developing the hypotheses of the project inasmuch these dimensions presuppose a distance between oral speech and mathematical notations or inscriptions. Moreover, attending to these dimensions will also allow us to highlight the fact that actors shaped practices of spatial navigation through such notations and inscriptions in the course of cognitive work with them (whether this be in for reasoning or calculating), something that we argue might be a key dimension of symbolic work.

Giaquinto (2007) has explored the diagrammatic dimensions of “calculation” and “symbol manipulation,” analysing how visual aspects are used in such mathematical work. What is the history of these diagrammatic features and of the practices that bring them into play? Are they related to the tabular types of computation that we find in, for instance, Sanskrit sources (resp. Keller, Montelle, and Koolakudlu forthcoming)? These are questions that will be addressed on the basis of sources in, e.g., cuneiform, Chinese, Sanskrit, Arabic, and Latin, as well of modern mathematical sources, which all abound in notations and computations with such diagrammatic features. Indeed, whether we pick up at random a Chinese source such as Qin Jiushao’s 秦九韶 *Mathematical Writings in Nine Chapters* (數書九章, 1247) or an Arabic source such as Ibn al-Yāsamīn’s *Grafting of Opinions of the Work on Dust Figures*, composed in Maghreb in the twelfth century (Oaks 2007), the notation of numbers and computations using place-value decimal notations and notations for equations and for determining their roots were all viewed as “figures.” And accordingly, mathematical work with them put into play similar diagrammatic features. These remarks offer a perspective from which to examine, in particular, the relationship between these features as practiced with numbers written with a place-value notation and as practiced with other types of notations and/or inscriptions (Chemla 1996).

In our historical survey of the diagrammatic features of symbolic notations and of the practices with them, modern notations and practices with them will also be of interest for us. They were long recognized by mathematicians like David Hilbert, when he wrote in his famous 1900 address to the Paris congress: “The arithmetical symbols are written diagrams, and the geometrical figures are graphic formulas” (p. 259). We are interested in exploring the history of the shaping of such practices.

The research devoted to the diagrammatic features of mathematical notations and inscriptions will examine whether such features are essentially tied to practices of computation or whether we encounter them in other types of mathematical practice. If they do prove to be attached to computation, we will need to spell out the properties that these diagrammatic features impart to the practices of computation that rely on such notations and/or inscriptions. In particular, a key question will be to understand the relationship that can be established between the deployment of notations and inscriptions with diagrammatic features and the use of place-value notations. The symbolic work enabled by diagrammatic

features of specific mathematical practices—the use of navigation they enable, and how this relates to prescription, computation and reasoning— will be central to this year’s work.

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Programme

6 January 2026, in JCMB, room 5323

Jens Høyrup (Roskilde University, emeritus)

Diagrams in Fibonacci's Extant Works

Abstract: Using a tolerant understanding of what constitutes a diagram, a first section of the talk looks at the diagrams used in Ibn al-Banna's *Talkhīṣ* and al-Qalaṣādī's *Kaṣf*, which turn out to be restricted to the writing of fractions and composite fraction lines, and a few other instances linked to the Arabic idiom for fractions.

The bulk of the talk looks at Leonardo Fibonacci's use of diagrams. Apart from the notations for fractions and ascending continued fractions, he turns out not to borrow from the “learned” *madrasah* level of Maghreb mathematics. In all his works, he uses rectangular frames that superficially looks like diagrams but are simply indications that numbers are supposed to be written on a whitened board. Beyond that he uses single-line lettered diagrams and Euclidean-style lettered diagrams; most of these show by their lettering to be borrowed from possibly Greek but at least mostly Arabic sources or earlier Latin translations. When geometric objects or situations are illustrated, they are also mostly diagram-like with no naturalistic pretensions. None of this is sensational, and it is comprehensible that Fibonacci's diagrams have attracted little attention.

In the end, lettering of Fibonacci's diagrams is taken up in detail, giving substance to the claim that it is informative about Fibonacci's sources and his use of them.

Roy Wagner (ETH, Zürich)

Is the mathematical running text a diagram?

Abstract: This year's focus is on the independence of mathematical notation from spoken language and on non-linear aspects of mathematical notation. While the running text encountered in premodern manuscripts often appears to be linear and to have a close correspondence with straightforward reading, the actual practices of engaging with mathematical texts may deviate from this appearance. In my rather speculative intervention, I will try to show in what ways a linear mathematical running text might be viewed as a diagram.

I will start with my own reading practices of mathematical texts as a historian of mathematics reading foreign languages in which I have a limited working knowledge. Using Malayalam and/or Italian examples, I will focus on my philological practices of breaking up the text, identifying key anchors and repeated “formulaic” expressions (in the sense defined by Reviel Netz in *The Shaping of Deduction*), and resolving embedded syntactical structures. During this analysis, I sometimes reorganize the text graphically and, in some sense, turn it into a diagram. Sometimes, I accompany the reading with ad-hoc somewhat diagrammatic symbolic annotations. I will then raise the question: to what extent can such practices be attributed to contemporary readers of mathematical texts? Our limited knowledge of learning and research practices will allow me to offer only tentative guesses.

If time permits, I will also consider examples of the non-linear interactions between a running Hebrew text dealing with operations on fractions and an apparently idiosyncratic calculation diagram that accompanies the text in the original manuscript.

3 February 2026, room 2901

Marcus Giaquinto (University College London, Emeritus)

A problem about matrices

Abstract: This talk concerns a question about the matrices of linear algebra: What, strictly speaking, are they? This is not a practical problem. If you are familiar with matrices (as I assume), you can recognise presentations of them and manipulate them to solve problems. But there are conflicting views among mathematicians about what constitutes a matrix, as revealed by a survey of their definitions. The problem is philosophical. In what follows I examine Alexander Paseau's attempt to resolve the conflict and to answer the question in his 2016 paper "Philosophy of the Matrix." *Philosophia Mathematica* (III) 25(2) 246-267. (Recommended reading, but not needed for the talk). While initially in agreement with Paseau's proposed answer and favourably impressed by his case, I became increasingly dissatisfied over time. I will give my reasons for dissatisfaction, argue for an alternative answer, and propose a wider view of mathematical entities than is currently standard.

Commentator Célestin Xiaohan Zhou (Institute for the History of Natural Sciences, CAS, & School of Mathematics, The University of Edinburgh)

Karine Chemla (School of Mathematics, The University of Edinburgh, & SPHERE)

What is at stake in diagrammatic features of written practices? Views based on Arabic and Chinese sources

Abstract: The talk begins with a discussion of Marcus Giaquinto's book *Visual Thinking in Mathematics* (2007, Oxford), and in particular with the chapter dealing with calculation and symbolic manipulation. I rely on Arabic and Chinese sources to highlight that they already show evidence of certain benefits associated with symbolic manipulation in the context of arithmetic computation.

Commentator Zehra Bilgin (School of Mathematics, University of Edinburgh & Institute for the History of Science, Istanbul Medeniyet University)

March 10, 2026, JCMB, room 5323

Jeffrey A. Oaks (Department of Mathematics and Computer Science, University of Indianapolis)

A diachronic look at diagrams

Abstract: In this talk I will show a variety of examples of diagrams from arithmetic and algebra books in medieval Arabic, Latin, and Italian, and continuing through sixteenth-century Europe. In these books we find three kinds: some diagrams show one step in a process, such as in the multiplication of two

numbers expressed in Indian notation; some express a relation, like an algebraic equation; and still others express a mathematical object, whether a fraction, a composite irrational root, an algebraic polynomial, or an amount expressed in terms of more than one denomination (like degrees, minutes, seconds or days, hours, minutes). There just might be enough in the examples I will show to begin thinking about continuity and differences in practice, especially as the techniques of calculation passed from Islamicate countries to Europe.

Commentator: Agathe Keller (LTE, Paris Observatory)

Taro Tokutake (SPHERE, Université Paris Cité)

Numerical Configurations with Diagrammatic Features in an Anonymous Commentary on Śrīdhara's [Book] of Three Hundred [Verses] (Triśatī): Algorithms for Extracting Square and Cube Roots

Abstract: In this presentation, I aim to explore numerical configurations with diagrammatic features found in algorithms for extracting square and cube roots as presented in the *Exposition on the [Book] of Three Hundred [Verses]* (*Triśatībhāṣya*). Śrīdhara, a mathematician who flourished in India around 800 CE, composed a Sanskrit arithmetic treatise, the *[Book] of Three Hundred [Verses]* (*Triśatī*). Over thirty manuscripts of this treatise have been handed down to us, and some of them are accompanied by commentaries. The *Exposition on the [Book] of Three Hundred [Verses]* is one such commentary. This commentary survives only in a single complete manuscript (LD Institute, Ahmedabad, No. 1559), and no information about its date, place, or author is found there.

As many Sanskrit authors do, Śrīdhara presents the algorithms for extracting square and cube roots in a versified form and briefly outlines only their core steps. The commentator, on the other hand, interprets the algorithms given by Śrīdhara and presents the calculation procedures step by step, sometimes offering numerical configurations and sometimes not. These configurations indicate an intermediate arrangement of digits in the course of the algorithms, which are executed on a working surface that exists outside the running text. Building on these observations, the following questions arise: Can diagrammatic features be perceived in the numerical configurations? How do these configurations function in the execution of the algorithms? And how do the configurations relate the running text to the practice involving them? To address these questions, I offer a detailed study of the algorithms for extracting square and cube roots on the basis of the *Exposition on the [Book] of Three Hundred [Verses]*.

Commentator: Alex Garnick (Harvard U. & SPHERE)

April 14, 2026, JCMB, room 5323

Célestin Xiaohan Zhou (Institute for the History of Natural Sciences, CAS, & School of Mathematics, The University of Edinburgh)

Written Calculation Practices and Numerical Notation in Fifteenth- and Sixteenth-Century Chinese Mathematical Texts

Abstract: In general narratives of the history of mathematics in China, scholars usually focus on the fact that during the fifteenth and sixteenth centuries computational tools were in a transitional phase, shifting from counting rods to the abacus. However, a closer examination of mathematical writings from this period reveals that, for operations such as multiplication and division that had previously relied on counting rods, there existed a diversity of methods for computation as well as for determining the order of magnitude of the results. Mathematical texts such as *Great Compendium of the Nine Chapters on Mathematical Methods with Analogies* (*Jiuzhang suanfa bilei daquan*, 1450) and *Unified Lineage of Mathematical Methods* (*Suanfa tongzong*, 1592) introduced distinctive computational approaches by means of writing practices, including procedures such as “calculation with magic squares” (*Hetu shushu*) and “written calculation” (*xiesuan*). The prescriptions and descriptions of these procedures in these works further reflect the ways in which the authors diagrammatized their written mathematical practices. In these contexts, what is the status and meaning of notations in the diagrammatic configurations? How is the value of place manifested within such computations? What distinctions and relationships exist between the numerical notations used in the diagram and the verbal explanations that assign meaning to numbers within the context? In what ways do these practices differ from and connect with earlier rod-calculation practices and later abacus practices? During the process of rendering operations into visual diagrams, which operational or instrumental elements were symbolically recorded and incorporated into printed texts? These questions constitute the focus of the present presentation.

Commentator Isobel Falconer (University of Saint-Andrews)

J.P. Ascher (University of Edinburgh, SIGMA UKRI-ERC Postdoc)

Typesetting Modern Algebra: The Modern Reception of Early Modern Contexts in Théotiste Lefèvre's Guide Pratique du Compositeur, 1855

Abstract: Algebraic typesetting has an uneven history. While Joseph Moxon describes typography as a mathematical art in 1683-4 after he had likely produced some of his own type for abstract symbol systems, he provides little specific guidance to setting mathematics of any sort. In 1755, John Smith explains that most compositors dislike setting algebra because the authors invent their own symbols, are very particular, and don't understand the difficulties printing. Théotiste Lefèvre's manual of 1855, then, is one of the earliest systematic, published accounts of setting mathematics for a larger community of printers. But even more importantly, Théodore de Vinne sponsors a translation of Lefèvre by Henry Fine for his epoch-making manual of 1904 rather than writing a new section on algebraic typesetting. Lefèvre's approach, then, appears to become standard in the anglophone and francophone world, for example influencing the 1962 American Mathematical Society's *Manual for Authors*.

This paper will present Lefèvre both retrospectively and prospectively: examining the particular early modern practices for typesetting that he aimed to regularise and showing how those practices became part of the look of modern algebra. In situating the practices chronologically, the paper attempts to think about how now-invisible, taken-for-granted aspects of modern mathematical symbolism can be investigated and articulated backwards. Influence is difficult to trace and this

paper proposes to reflect on approaches to influence as well as laying out the historical facts.

Commentator Aileen Fyfe (University of Saint-Andrews)

May 5, 2026, JCMB, room 5323

Change of program

Arilès Remaki (Mainz Universität)

Writing the composition: Linear vs tree-like. Examples in Leibniz

Abstract: *tba*

Karine Chemla (School of Mathematics, The University of Edinburgh, & SPHERE)

What is at stake in diagrammatic features of written practices? Views based on Arabic and Chinese sources (Part II)

Abstract: The talk begins with a discussion of Marcus Giaquinto's book *Visual Thinking in Mathematics* (2007, Oxford), and in particular with the chapter dealing with calculation and symbolic manipulation. I rely on Arabic and Chinese sources to highlight that they already show evidence of certain benefits associated with symbolic manipulation in the context of arithmetic computation.

Commentator: Agathe Keller (LTE, Paris Observatory)

Ivahn Smadja (Nantes Université & Institut Universitaire de France)

Iterated exponentials, powers and iterates: Which diagrammatization? Which symbolism?

Abstract: This exploratory contribution aims at analyzing a significant shift around 1800 in the way some diagrammatic features of mathematical symbolism were addressed, conceived, and exploited in mathematical practice. As a guiding thread into the thick of it, we will first focus on iterated exponentials, tracing the history of the topic from Euler to the British mathematicians of the Analytical Society, and beyond. A broader purpose, however, is to show how this perplexing issue connects with new ways to think about diagrammatization and symbolism, increasingly brought to the fore in the early decades of the nineteenth century.

While commenting on the striking analogy between the Bodhisattva's calculus in the *Lalitavistara Sūtra* and Archimedes' *Sand Reckoner*, in his *Mémoire sur la propagation des chiffres indiens* (1863), historian of mathematics Franz Woepcke made an interesting observation, suggesting a watershed in the history of symbolism. The numbers attested in these ancient sources are expressed by means of extended scales of numerical names. They may seem huge, but, Woepcke argued, they are far from marking "the limit of the possible in terms of notation". He hinted at his own mathematical work on iterated exponentials, taking up and clarifying issues that arose decades earlier. These debates hinged on the role of nested parentheses in defining iterated operations, the status of such symbolic expressions as

$$\left(\left((a^a)^a \right)^{a \cdots a} \right)^a \quad \text{or} \quad a^{\left(a^{\left(a^{\left(a^a \right)} \right)} \right)},$$

and further the possibility of constructing an open-ended hierarchy of operations. Unfolding this sequence will lead us to look deeper into the meaning and scope of Arbogast's so-called "method of separation of symbols", thoroughly discussed by the British Analysts.

Commentator David Waszek (ENS Ulm, Paris)

June 2, 2026, room 5323

Zehra Bilgin (School of Mathematics, University of Edinburgh & Institute for the History of Science, Istanbul Medeniyet University)

Solving algebraic equations with tables: Examples from Jamshīd al-Kāshī's Miftāḥ al-ḥisāb

Abstract: Jamshīd al-Kāshī's (d. 1429) work *Miftāḥ al-ḥisāb* (or *Miftāḥ al-ḥussāb*) stands as one of the most significant mathematical treatises written in Arabic in the fifteenth century. Having been extensively studied across the Persian and Ottoman worlds, it functioned as a canonical mathematical text for centuries. This work provides a comprehensive synthesis of the arithmetic, algebra, and mensuration knowledge of its era, distinguished particularly by its systematic use of diverse tables to address a wide array of problems.

Notably, the treatise includes a dedicated chapter addressing various complex algebraic problems of several kinds. The functional scope of the tables within this section ranges from the initial presentation of a problem to its final resolution, including the verification of operations and the display of alternative methods. Al-Kāshī himself underscored the utility of these tabular structures in his introduction, signalling their importance to his pedagogical and mathematical approach. In this presentation, by examining examples of these problems and their solutions through various tabular formats, I will offer an analysis of the diagrammatic features of these tables and their role in solving algebraic problems.

Commentator Prof. Qu Anjing (Northwest University, Xi'an)

Tanaka Noriko 田中紀子 (Department of Education for Human Growth, Naragakuen University)

Mathematical Expressions by school in Japan During the Edo Period

Abstract: During the Edo period (1603-1867) there were several schools of mathematics in Japan. The most famous and popular school was the Seki school [関流] of the renowned mathematician Seki Takakazu [関孝和] (?-1708). In addition to the Seki school, there were various other schools such as Takuma school [宅間流], Shisei-Sanka school [至誠賛化流], Saijyo school [最上流], Miike school [三池流] etc.

Mathematical texts were written vertically, and the methods of mathematical expression also differed according to the school. The Seki school's way of expressing mathematics is inspired by the motif of *Sangi* [算木] (arithmetic sticks), which was introduced from

China to Japan. The *Sangi* were placed vertically for the first, hundred, and ten thousand places and horizontally for the tenth, thousand, and hundred thousand places. The red blocks represent positive numbers, and the black blocks represent negative numbers. When writing in books, negative numbers were represented by diagonal lines to express them in black ink only. Seki Takakazu developed *Bō-Shohō* [傍書法], which enabled the representation of both numbers and letters within a single formula, thereby improving the expression of mathematical equations.

Saijyo school was a school founded by Aida Yasuaki [会田安明](1747-1817). The Saijyo school's way of writing formula is almost identical to that of Seki school. Aida continued to criticise the Seki school, yet his mathematical expressions showed little originality. Shisei-Sanka school was a school founded by Hurukawa Ujikiyo [古川氏清](1758-1820). The expression of the formula is almost the same as the expression of Seki school's formula. This school has preserved a rare book entitled "Kiou Collection" [淇澳集], which details methods for studying Japanese mathematics. This book reveals that mathematics was kept secret within each school.

Takuma school was a school founded by Takuma Yoshiaki [宅間能清] in Osaka. Takuma school does not use numerical expressions of arithmetic sticks. Negative numbers are written as *fu* (フ) in Japanese *katakana* [カタカナ], and numbers are one (一), two (二), three (三) in Japanese *kanji* [漢字].

The mathematics of Takuma School involves complex factorization, which we believe this is made possible by the notation of the School.

The Miike School was founded by Miike Ichibei [三池市兵衛]. The algebraic notation of the Miike School possesses a special form. A hook is placed above the vertical line. When there are numerous positive and negative terms respectively, these are grouped separately; negative terms are indicated by drawing a diagonal line through the vertical line.

Furthermore a license was required to teach Japanese mathematics in the Edo period. Licences varied by school, and those who received them were able to teach mathematics. The Seki school was the largest, but whether due to the secrecy surrounding Japanese mathematics, a nationwide standardisation of notation was never achieved. I would like to deepen my consideration of the spread of the schools of mathematics during the Edo period and why the Seki school could not unify all the schools.

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Tanaka Noriko [田中紀子]. Looking at Takuma School's Collection of exquisite formulae and Methods of transformation of equations from an educational point of view. *RIMS Kōkyūroku Bessatsu* B92, 151-166. 2023.

Commentator Taro Tokutake (JSPS and SPHERE, CNRS & Univ. Paris Cité)