THE UNIVERSITY of EDINBURGH
School of Mathematics

Some Hints and Answers

Below are hints on which players have winning strategies on the ten games

Game 1: Cut has a winning strategy

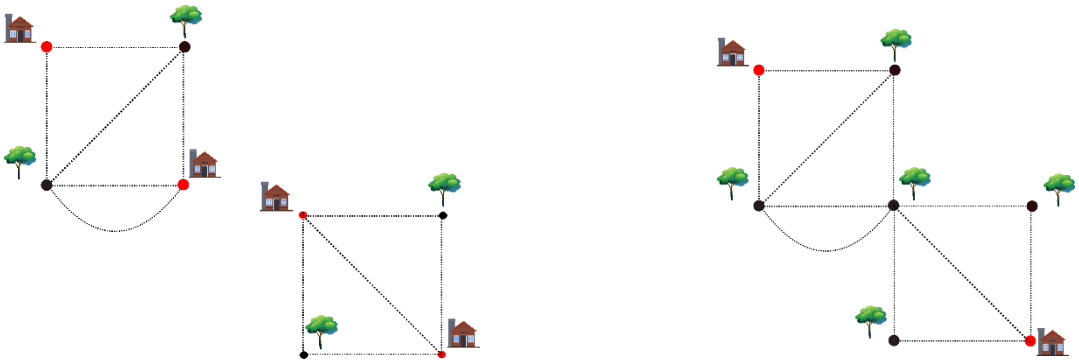
Game 2: The player who starts has a winning strategy, provided they take the diagonal line

Game 3: Join has a winning strategy. Video 3 will explore some strategies in depth

Game 4: The player who starts has a winning strategy; a good first move is to take the bottom diagonal. Video 4 will explain why, and will explain how a mathematician would approach the game.

The core idea is that Diagram 4 is a combination of diagram 3 (top left) and diagram 2 (bottom right). Compare the pictures below.

We know (from video 2 and 3) that on Game 3, Join has a winning strategy, while on Game 2, the first player wins if they take the diagonal.



Hence, if Join takes the bottom diagonal as a first move, then they only need to worry about the top left diagram, which is basically Game 3, where they have a winning strategy. If Cut takes the bottom diagonal as a first move, then they can easily cut off the bottom house within a couple of moves.

This logic process is something mathematicians often adopt. When they see a complicated problem they cannot solve (Game 4 in this case) they try to break it down into simpler problems they already know how to solve (Games 2 and 3 in this case).

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Game 5: Join has a winning strategy. Video 5 will explore some strategies in depth.

Game 6: The player who starts has a winning strategy. We note that Game 6 is basically a combination of Game 5 and Game 2, and we reason in the same way as in video 4.

Game 7: Cut has a winning strategy

Game 8: Join has a winning strategy

Game 9: The player who starts has a winning strategy. A good move is to take the top diagonal first.

Game 10: Cut has a winning strategy

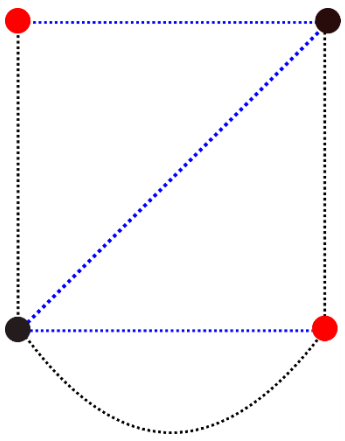
When Join has a winning strategy

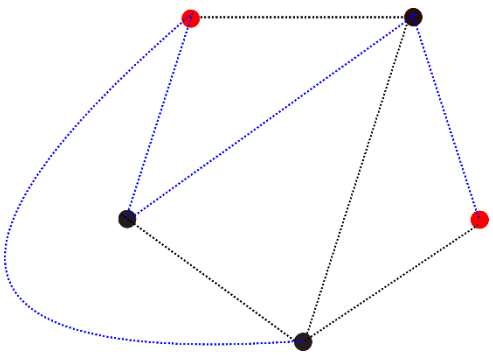
For anyone who is curious, below I explain how to understand whether Join has a winning strategy. The concepts touch on some advanced Mathematics: do not feel obliged to read this part. I start with some terminology.

In advanced mathematics, a **graph** is a collection of points joined together by lines. All the diagrams where we play the games are examples of graphs. A **tree** is a graph which has no circuits.

Join has a winning strategy on a particular diagram if the diagram contains two trees, which are connected, touch all points, but have no lines in common.

Below are examples: Game 3 and Game 5. The two trees are drawn in different colours (black and blue).





The above pictures show one way to draw these two trees, but this is not the only way.

Details are quite complicated, but the core idea is that whenever Cut deletes a line on one tree, Join would always have a way to recalculate a route using the other tree. However, when diagrams are big, it may be very difficult for Join to spot the right move. Hence they may lose anyway.

If you would like to explore further, you can have a look at this paper:

http://www.cs.cmu.edu/afs/cs/academic/class/15859-f01/www/notes/shannon.pdf

But be warned, this paper is not easy to read at all!