Building interest rate scenarios

ATRC 2014

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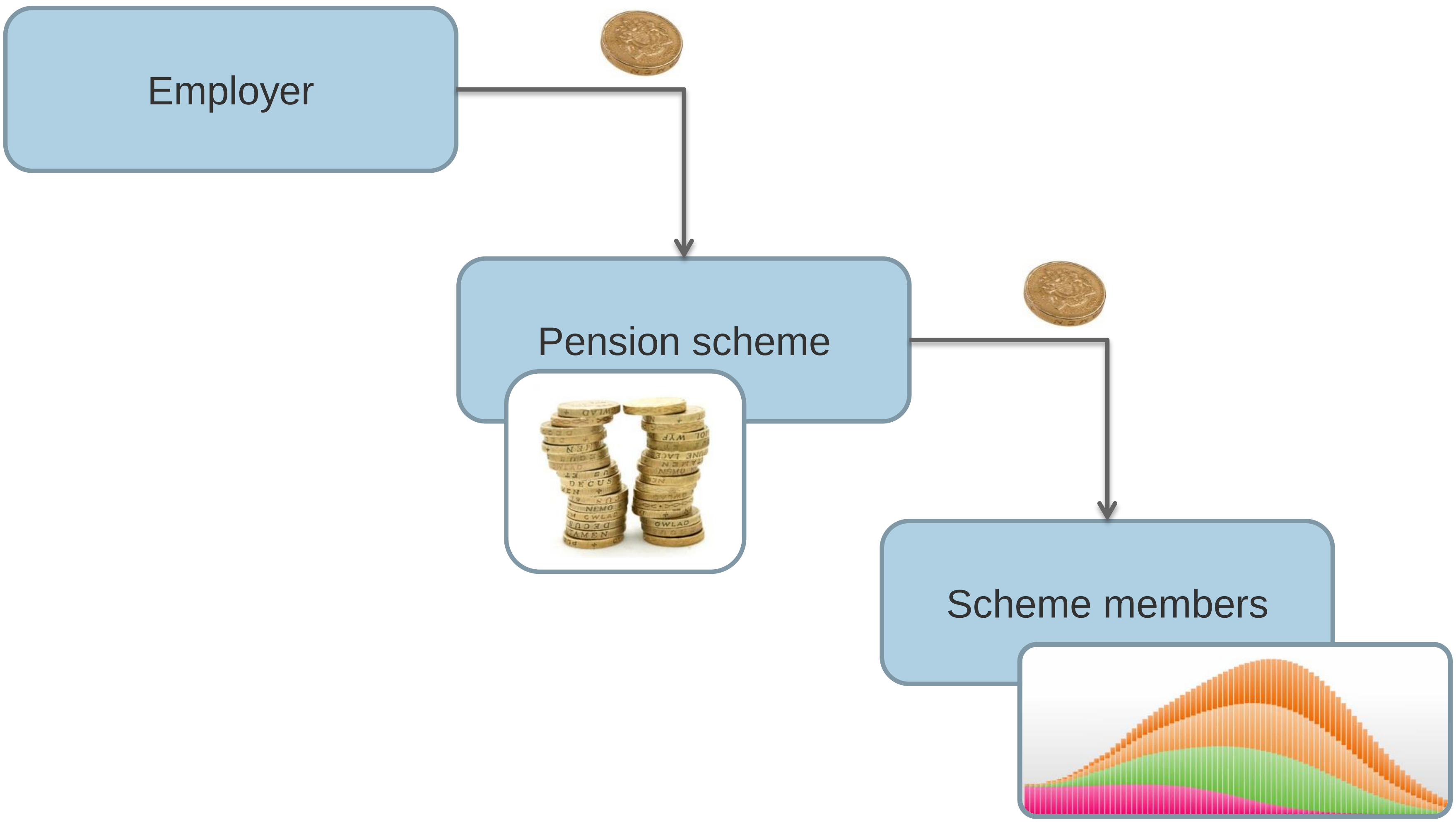
Risk management

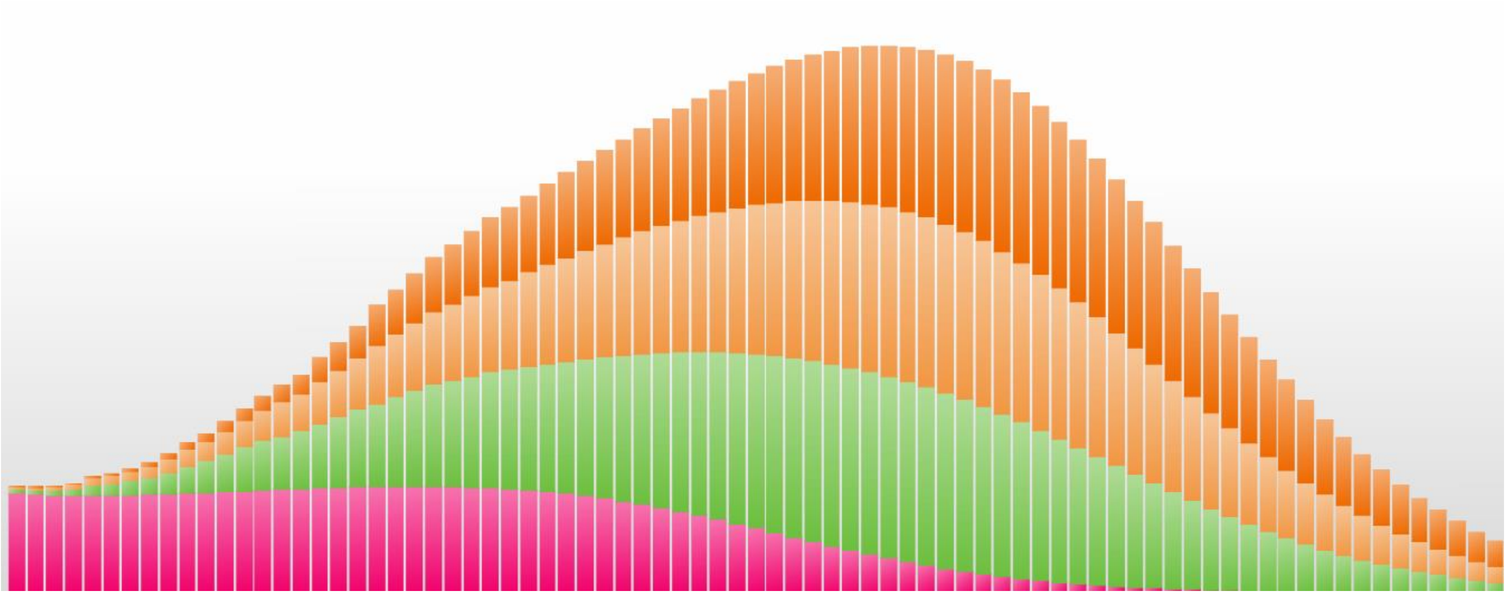
We’re in the business of risk-management

- On behalf of large groups of individuals
- Over very different time horizons

Pension scheme example



Pension scheme example



1. How much should be invested so that the probability of running out of cash is low enough?
2. What’s the market-consistent value of the liability?

Economic Scenarios

- Probability of running out of cash:

Real world probabilities

- Market consistent value of liability:

Risk neutral probabilities

Outline

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- Why use economic scenarios?
- Main challenges
- Empirical facts about interest rates and inflation
- Arbitrage free pricing
- Calibration

Main challenges

Main challenges

- Many choices
- Complexity
  - No analytical formulas
  - Computational resource constraints
    - Size of time-steps
    - Number of scenarios

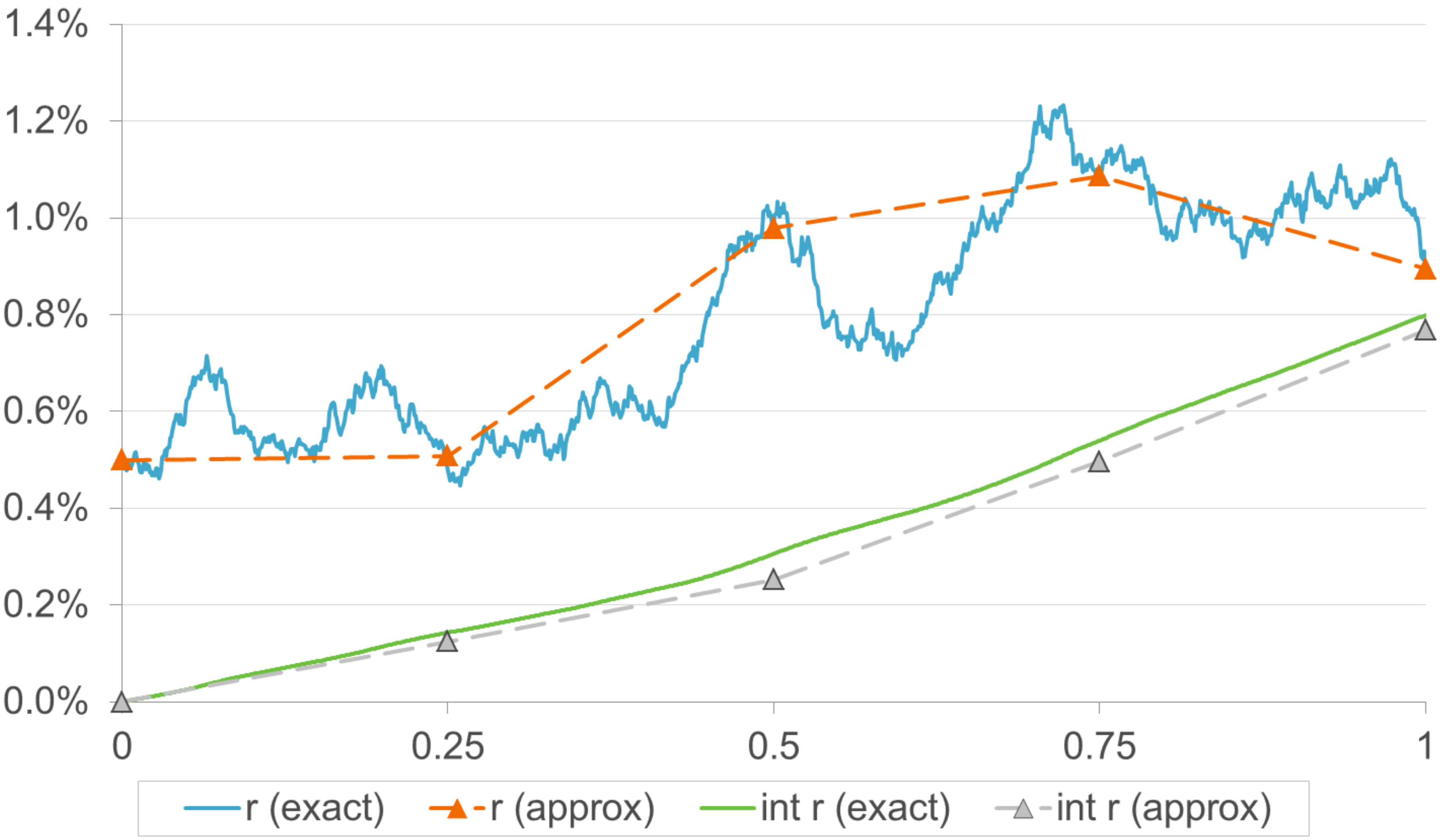
What are we trying to do?

Many choices

Risk factor	Initial Yield Curve	Interest Rate	Inflation	Credit	…
Model	Nelson-Siegel Cubic Spline Smith-Wilson …	Deterministic Vasicek G2++ …	Jarrow-Yildirim Stoch Volatility …	CIR++ JCIR++ JLT …	…
Calibration	Gilts Swaps OIS …	Swaptions Caps/Floors Distrib Targets …	YY Inf Swaps LPI Bonds Distrib Targets …	Corp Bonds CDS Spreads …	…

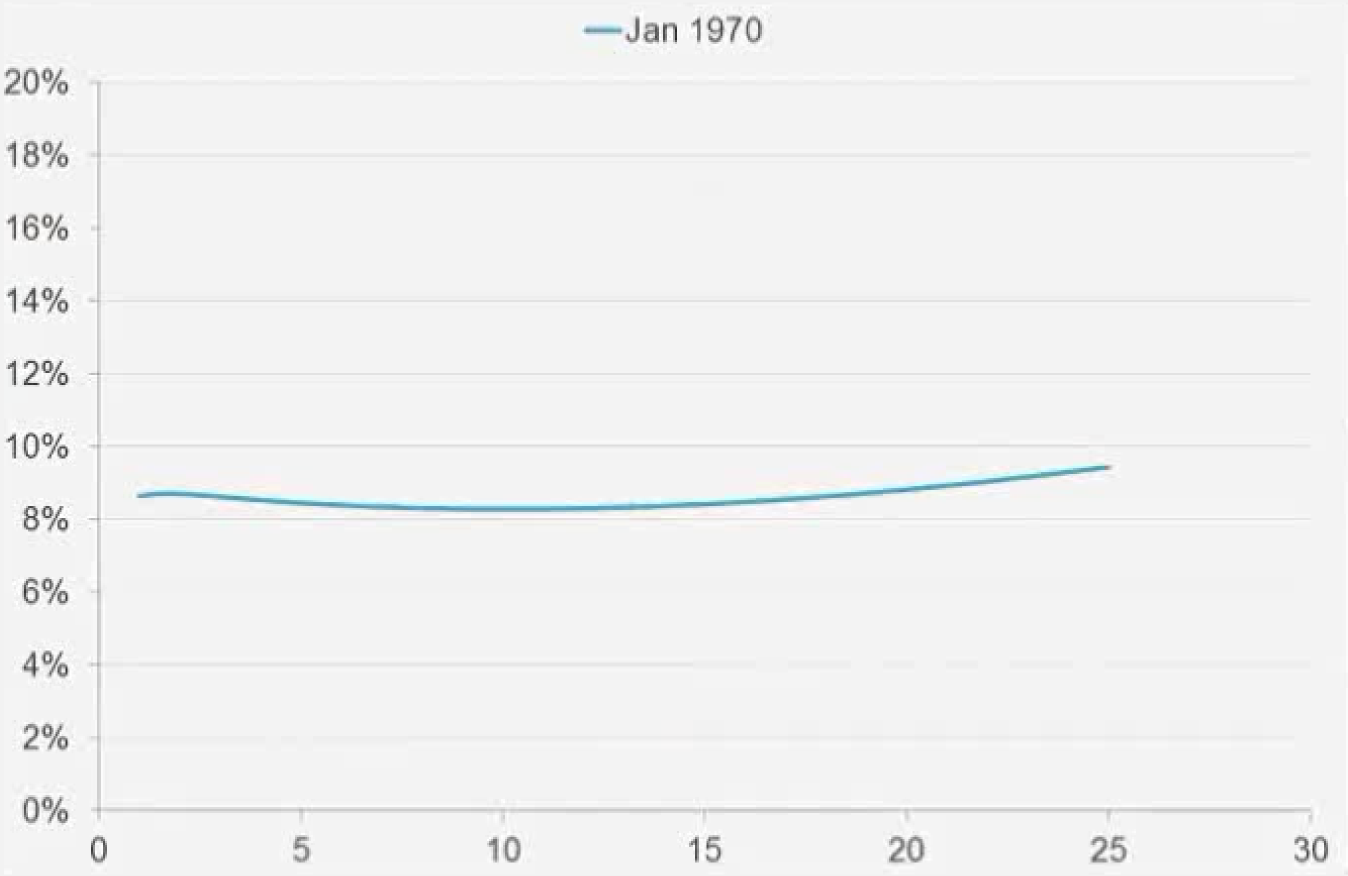
Dependencies

Size of time steps



Empirical facts about interest rates

A brief history of interest rates



A brief history of interest rates

1973


1981


1987


1992


1997


2001

2008


March 2009

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Gilt spot rates - BoE

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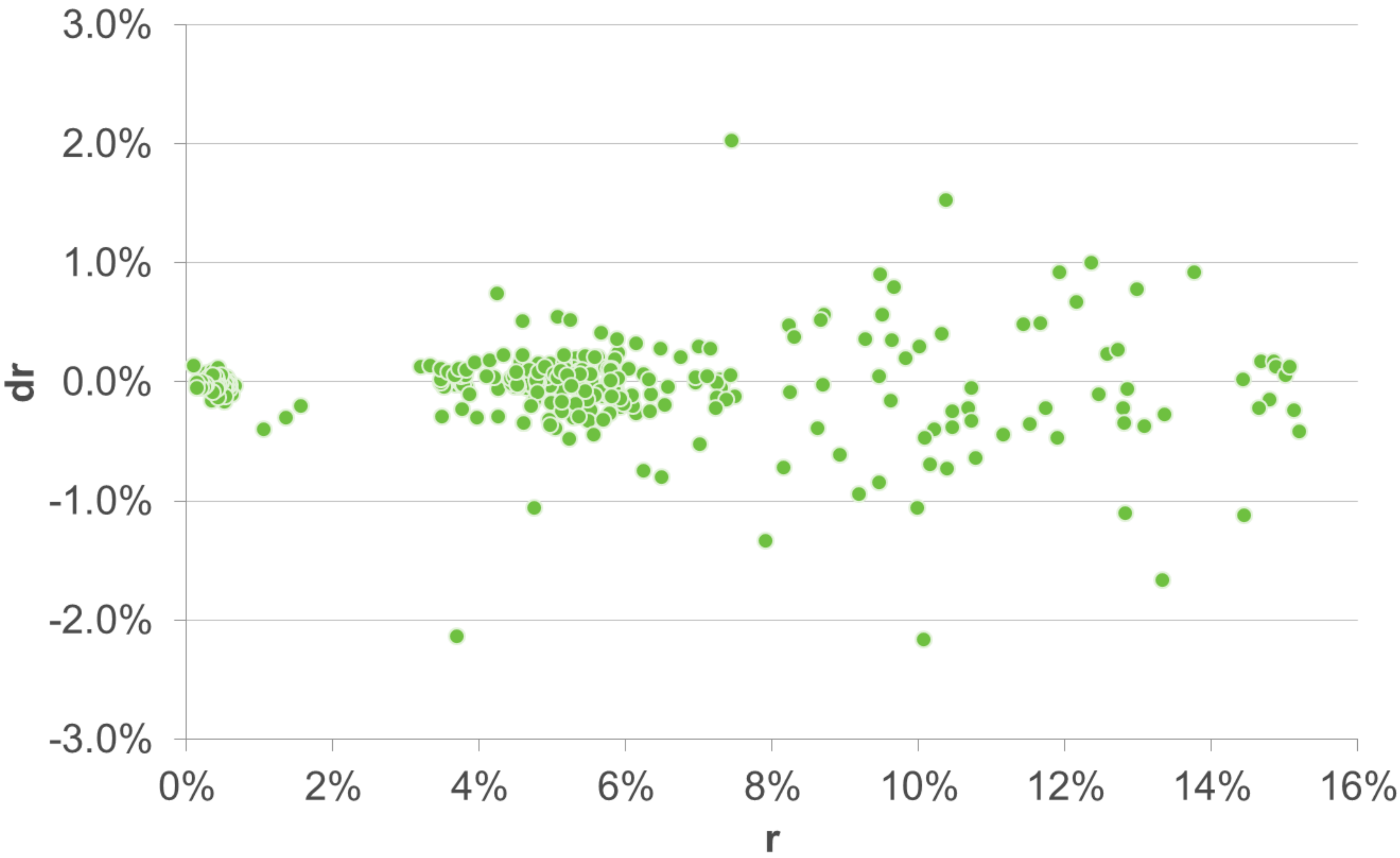
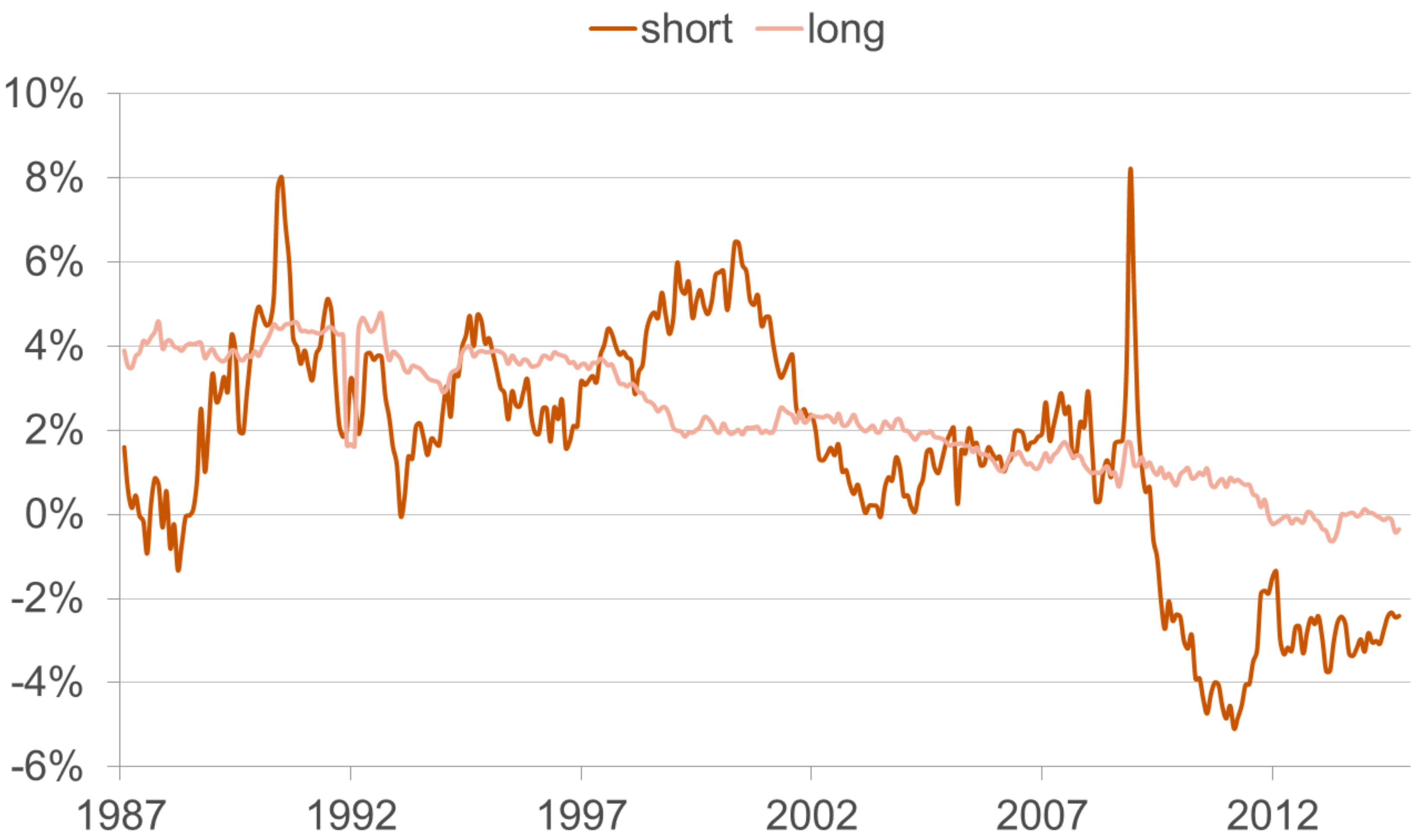
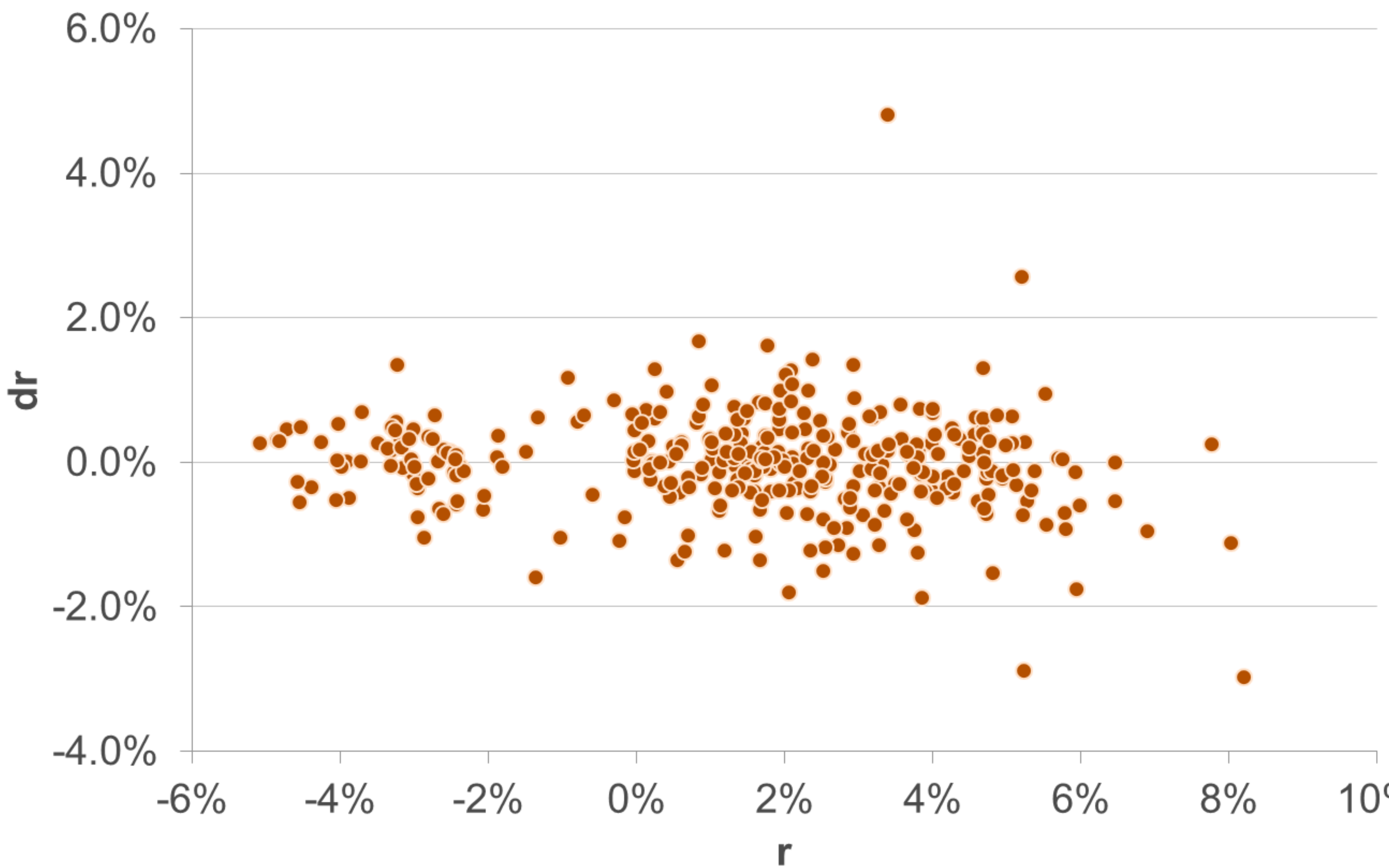
Nominal short rates



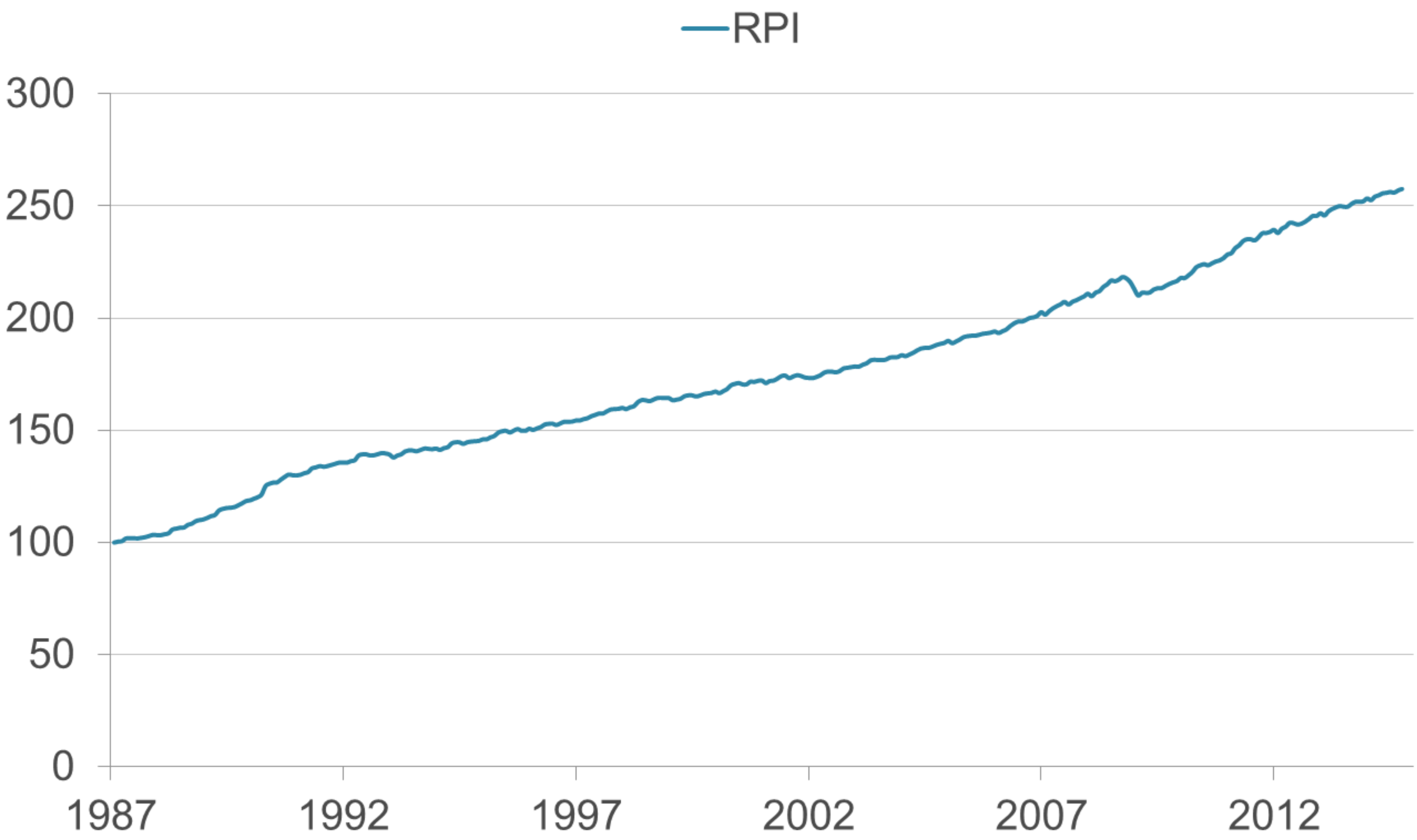
Real spot rates - BoE



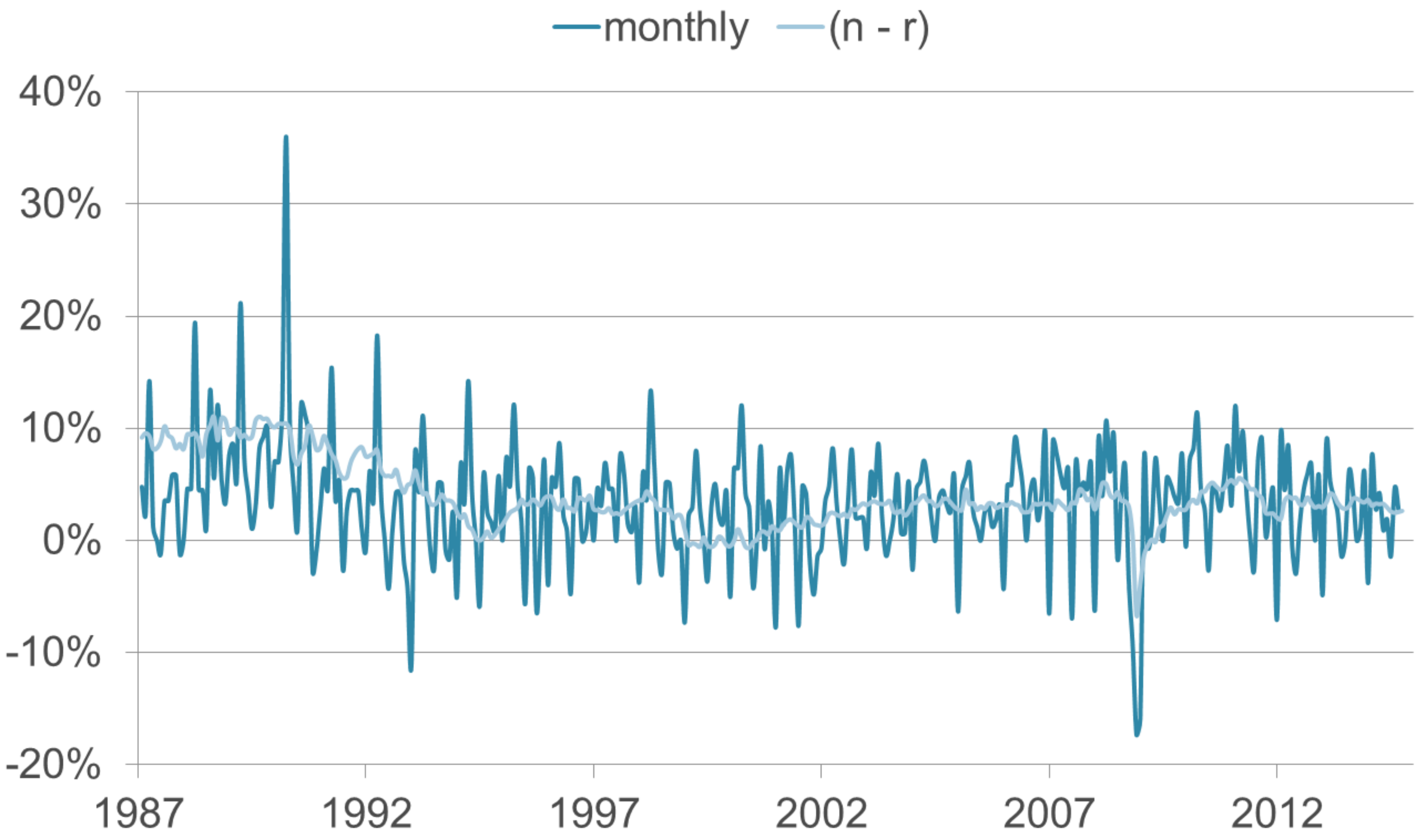
Real short rates



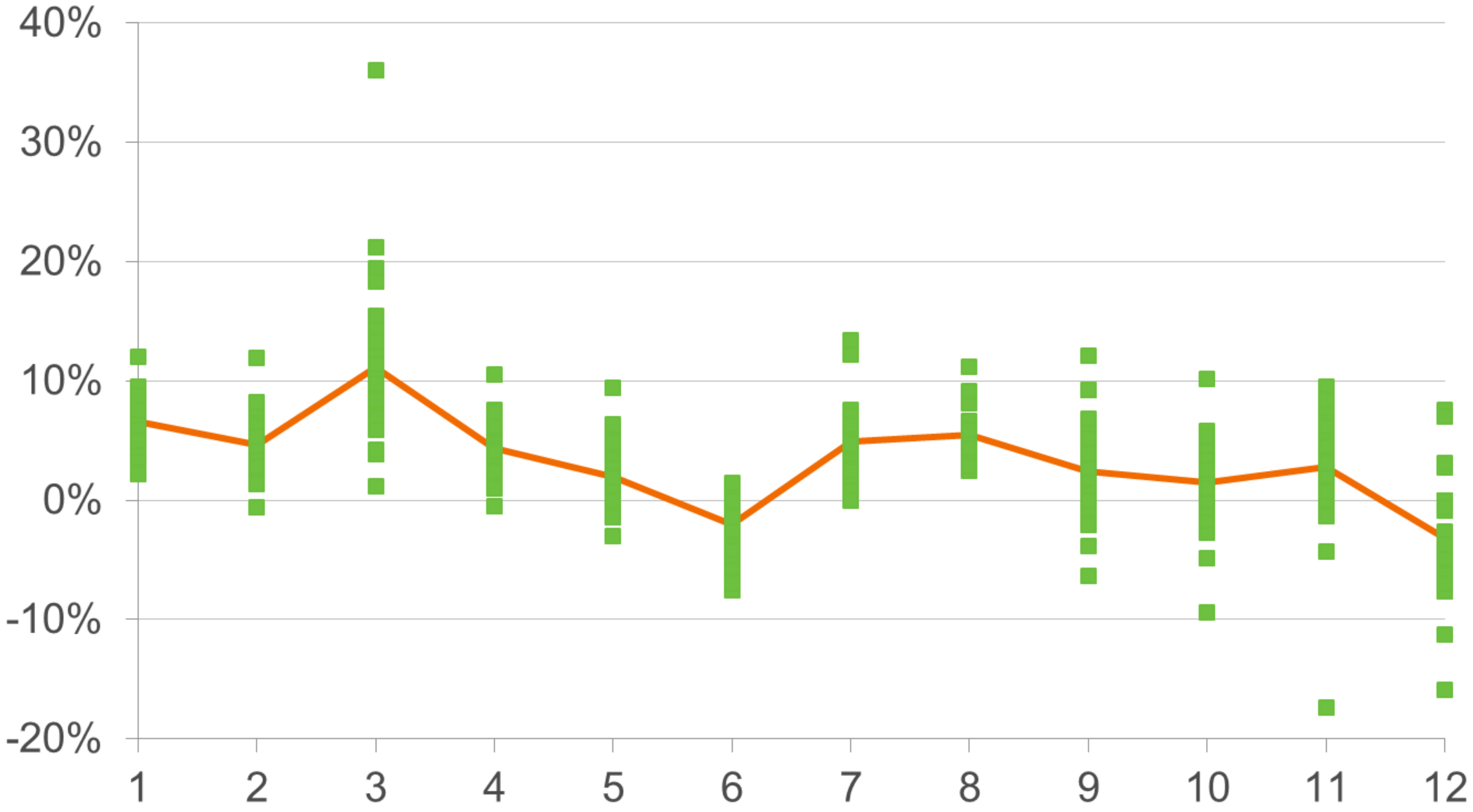
Inflation (RPI)



Inflation (RPI)



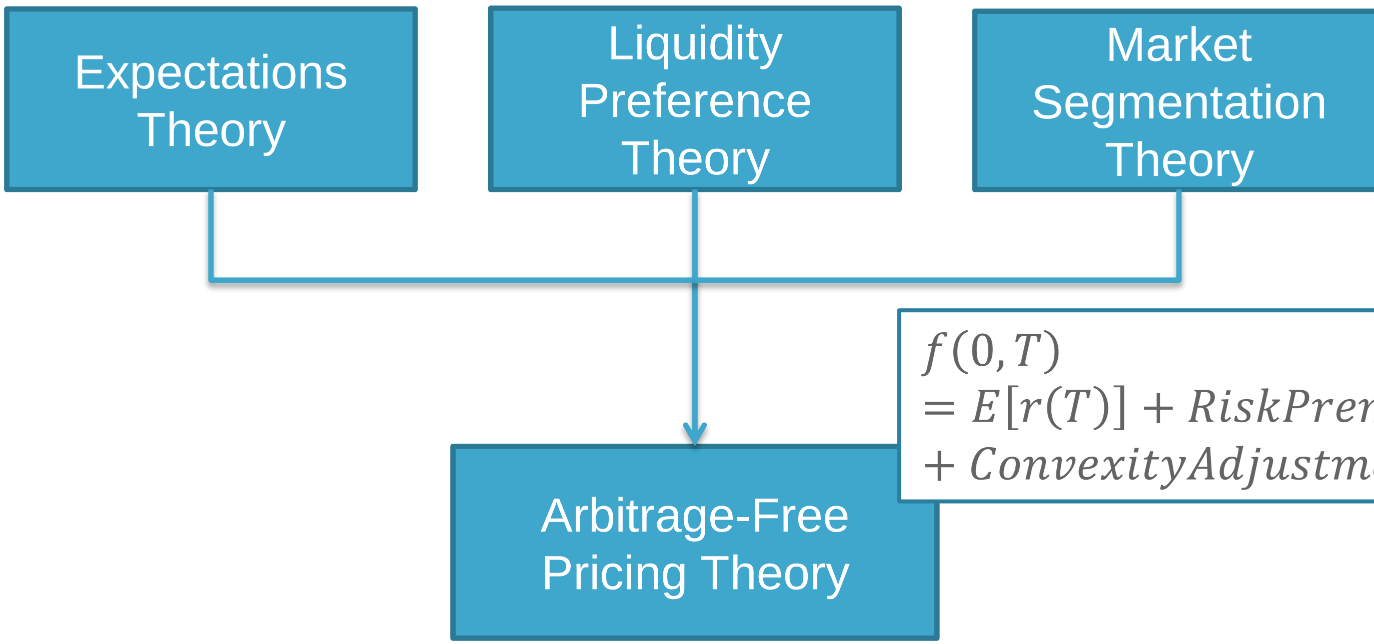
Inflation (RPI) by month



Facts about interest rates & inflation

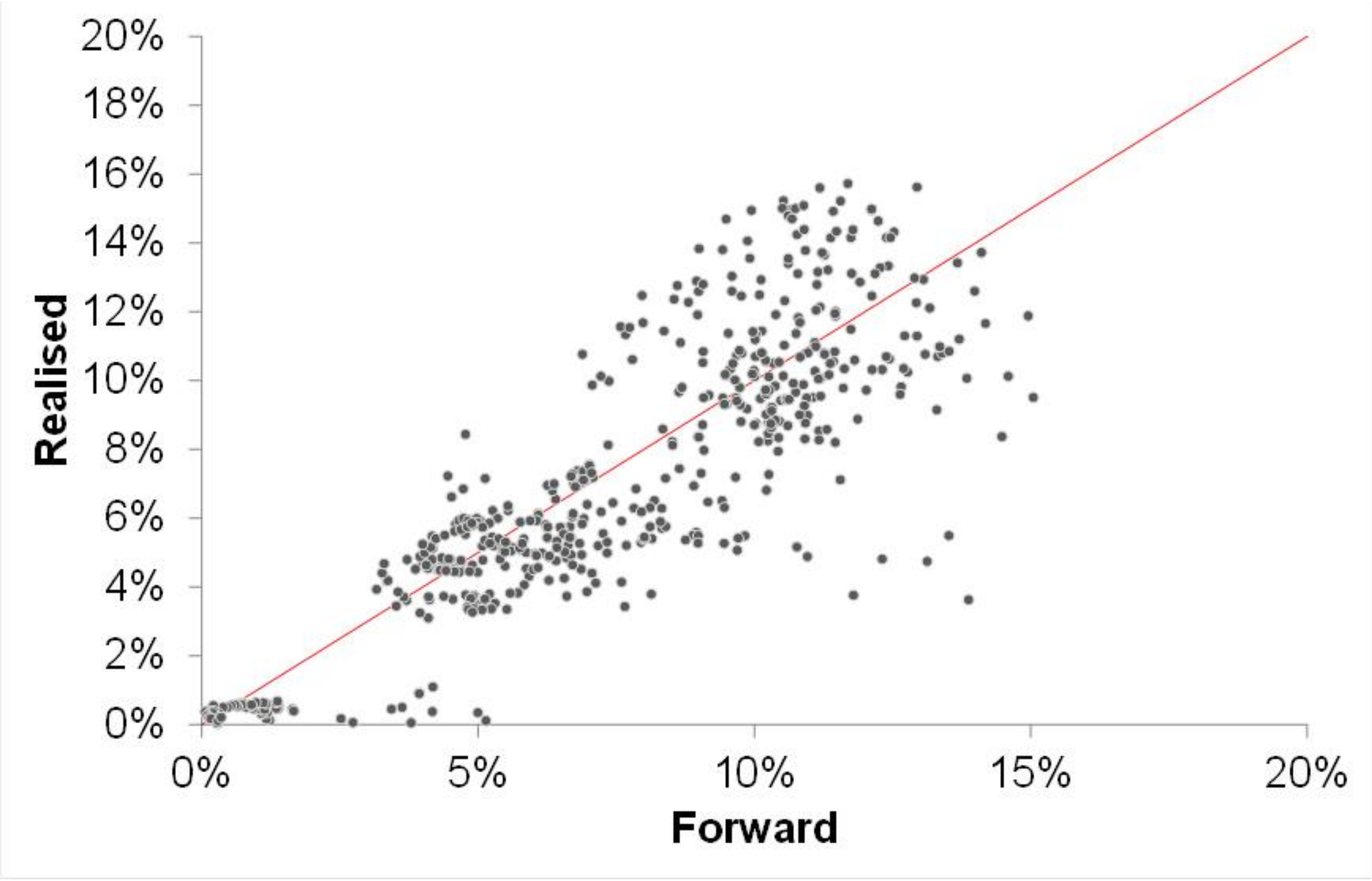
- Interest rates do not vary deterministically
- Nominal rates are (usually) positive, real rates can be negative
- They appear to be observations of a jump-diffusion process
  - Regime shifts
- The short rate is more volatile than the long rate
- Short and long rates exhibit decorrelation
- Seasonality in inflation

General theories of interest rates

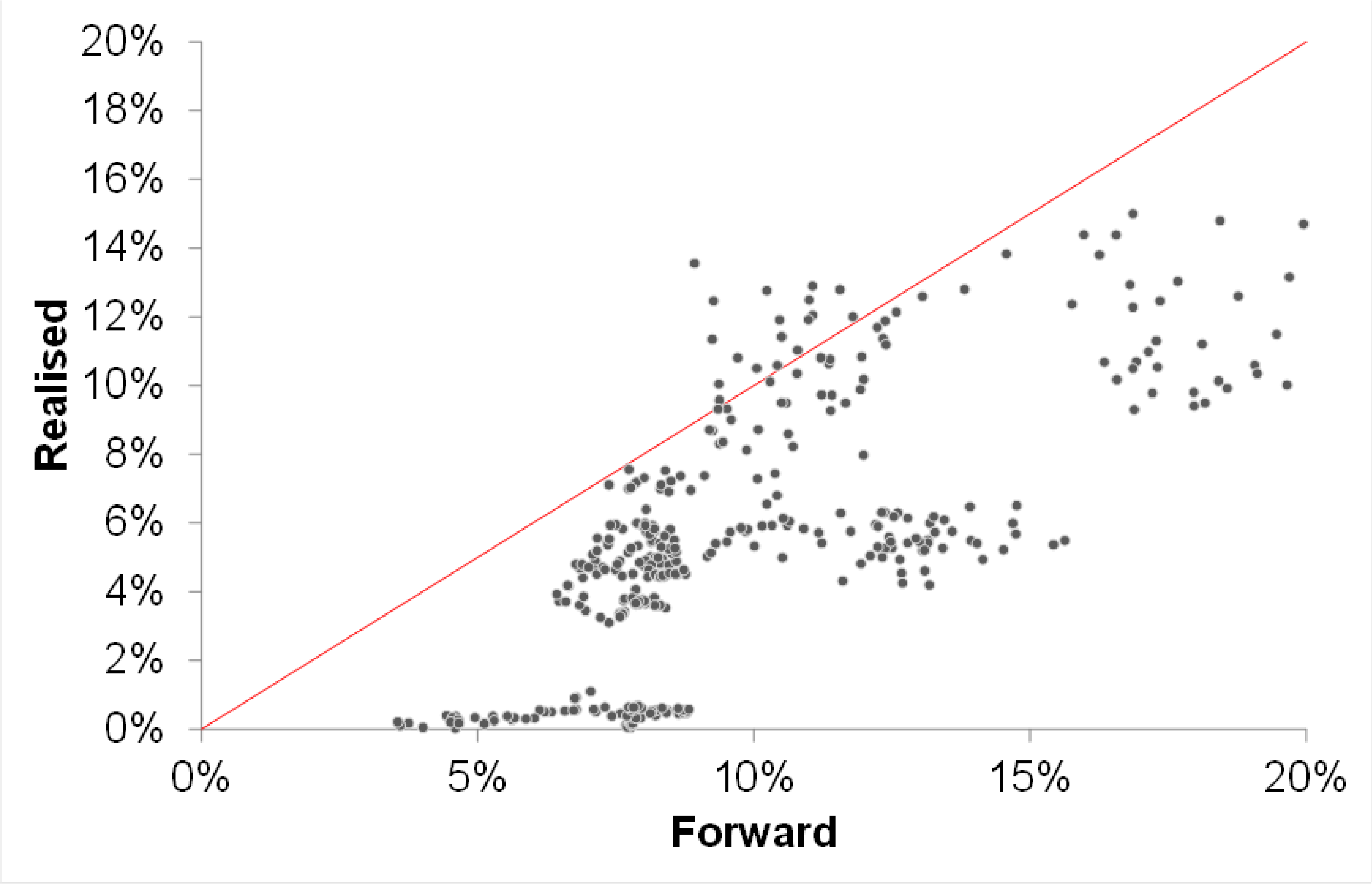


$$f(0,T) = E[r(T)] + RiskPremium + ConvexityAdjustment$$

Forward rates as expectations?



Forward rates as expectations?



Arbitrage free pricing

Arbitrage

- A model that allows for arbitrage leads to combinations of prices that do not make sense
- How do we build an arbitrage free model?

Fundamental Theorem of Asset Pricing

- Cash account: $B(s) = B(t)\exp\left(\int_t^s r(u)du\right)$

Arbitrage free dynamics $\Leftrightarrow$ Existence of Q ($\sim P$) such that, for $t \leq s \leq T$,

$$E_Q\left[\frac{P(s,T)}{B(s)}\,|\mathcal{F}_t\right] = \frac{P(t,T)}{B(t)}$$

Pricing

- It follows ($s = T$) that the price at time t of a derivative with payoff $V(T)$ at time T is

$$V(t) = E_Q[\underbrace{e^{-\int_t^T r(u)du}}_{\text{Stochastic discount factor } D(t,T)} V(T)|\mathcal{F}_t]$$

Stochastic discount factor $D(t,T)$

A word of caution

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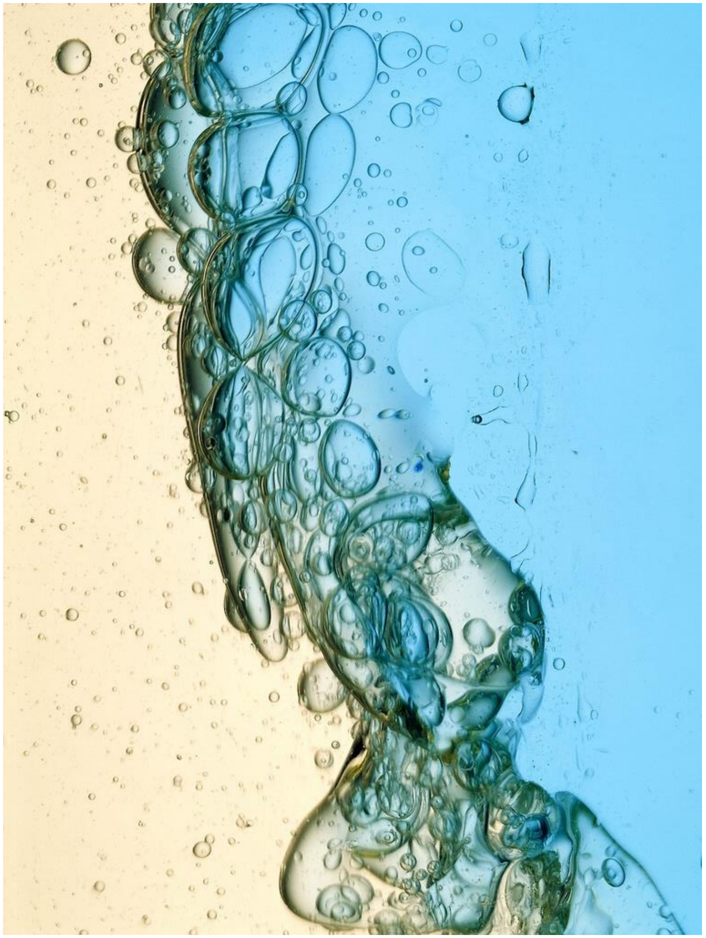
Our requirements so far

- Arbitrage-free pricing model
- Realistic probability distributions
- Capable of pricing interest rate and inflation derivatives



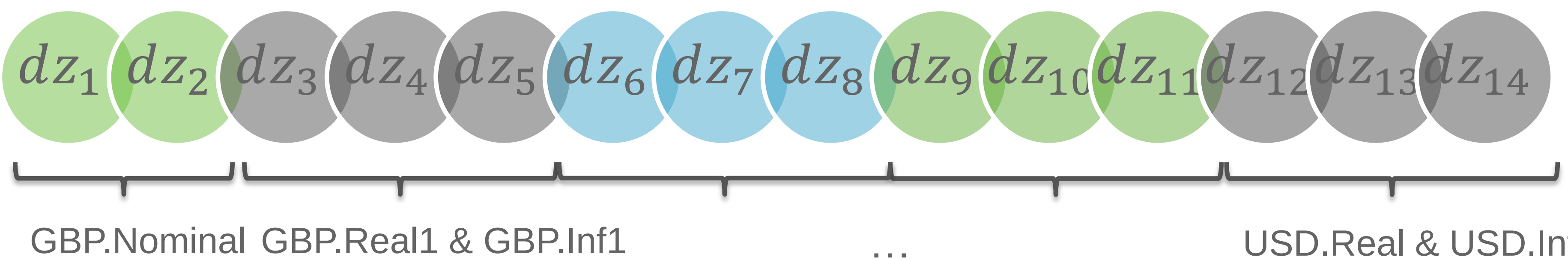
Real rates and inflation

- Foreign currency analogy
  - In each economy there are two currencies: the nominal (i.e. domestic) currency and the real (i.e. foreign) currency. The exchange rate between the two currencies is given by the inflation index $I(t)$ (e.g. CPI index): one unit of real currency is equal to $I(t)$ units of nominal currency.
- Nominal rates, real rates and inflation are modelled simultaneously

Modelling multiple economies

$$\begin{pmatrix} 1 & <dz_1, dz_2> & \dots & <dz_1, dz_{14}> \\ <dz_1, dz_2> & 1 & \dots & <dz_2, dz_{14}> \\ \vdots & \vdots & 1 & \vdots \\ <dz_1, dz_{14}> & <dz_2, dz_{14}> & \dots & 1 \end{pmatrix}$$

- GBP
  - Nominal: 2 factors
  - Real1(e.g. RPI): 2 factors; Inflation1: 1 factor
  - Real2 (e.g. CPI): 2 factors; Inflation2: 1 factor
- USD
  - Nominal: 2 factors; Exchange: 1 factor
  - Real: 2 factors; Inflation: 1 factor



Quanto adjustment (Risk-Neutral)

- $\frac{X(T)}{B(T)}$ is a martingale $\leftrightarrow \frac{dX}{X} = n\,dt + \cdots$
- $B^{\text{real}}(T)I(T)/B(T)$ is a martingale $\leftrightarrow \frac{dI}{I} = (n-r)dt + \cdots$
- $X^{\text{real}}(T)I(T)/B(T)$ is a martingale $\leftrightarrow \frac{dX^{\text{real}}}{X^{\text{real}}} = (r - \rho_{XI}\sigma_I\sigma_X)dt + \cdots$

Quanto adjustment

Model calibration

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Calibration

- Calibration of a model
  - Find the best choice of parameters such that the (weighted) squared difference between targets and corresponding model values is as small as possible

$$\Theta^2(\vec{p}) = \sum_{i=1}^{N} w_i \left(V_i^{\text{target}} - V_i^{\text{model}}(\vec{p})\right)^2$$

  - Example of targets:
    - Market prices of financial instruments (e.g. swaptions, inflation swaps, etc.)
    - Distributional properties (e.g. short rate dispersion in 3yrs, or long term dispersion of 17yr spot rate)

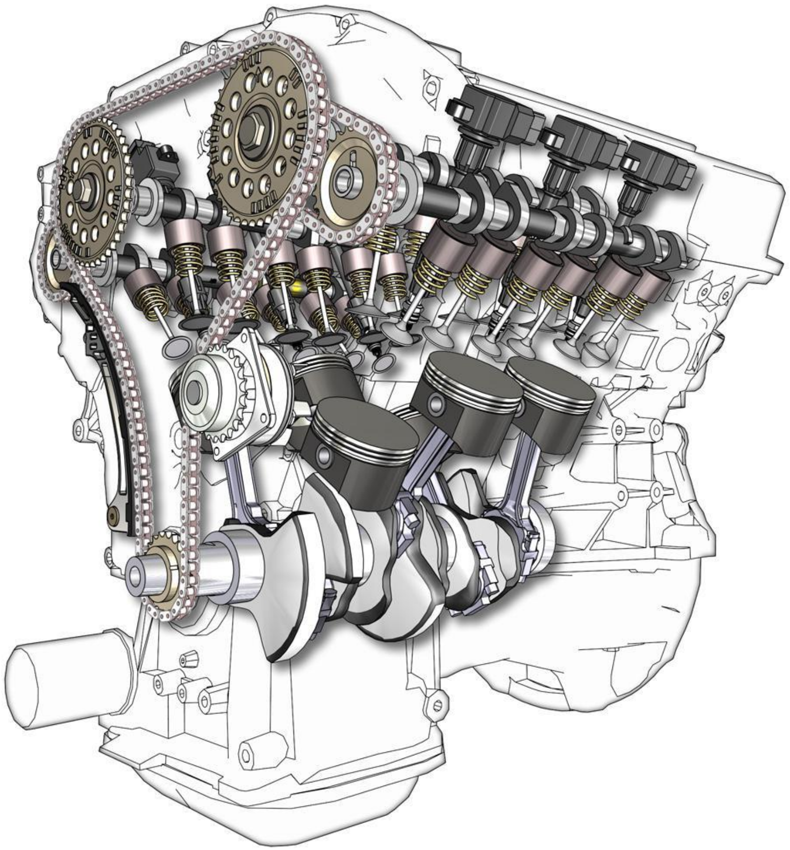
Calibration methods

- Cascade calibration
  1. Calibrate Nominal Short Rate Model
  2. Calibrate Real Rates & Inflation Model

- Full calibration
  - Calibrate Nominal Short Rate Model and Real Rates & Inflation Model as a single model

Calibration

- It is an non-linear least squares optimisation problem
- Can be solved numerically
  - Levenberg-Marquardt optimiser

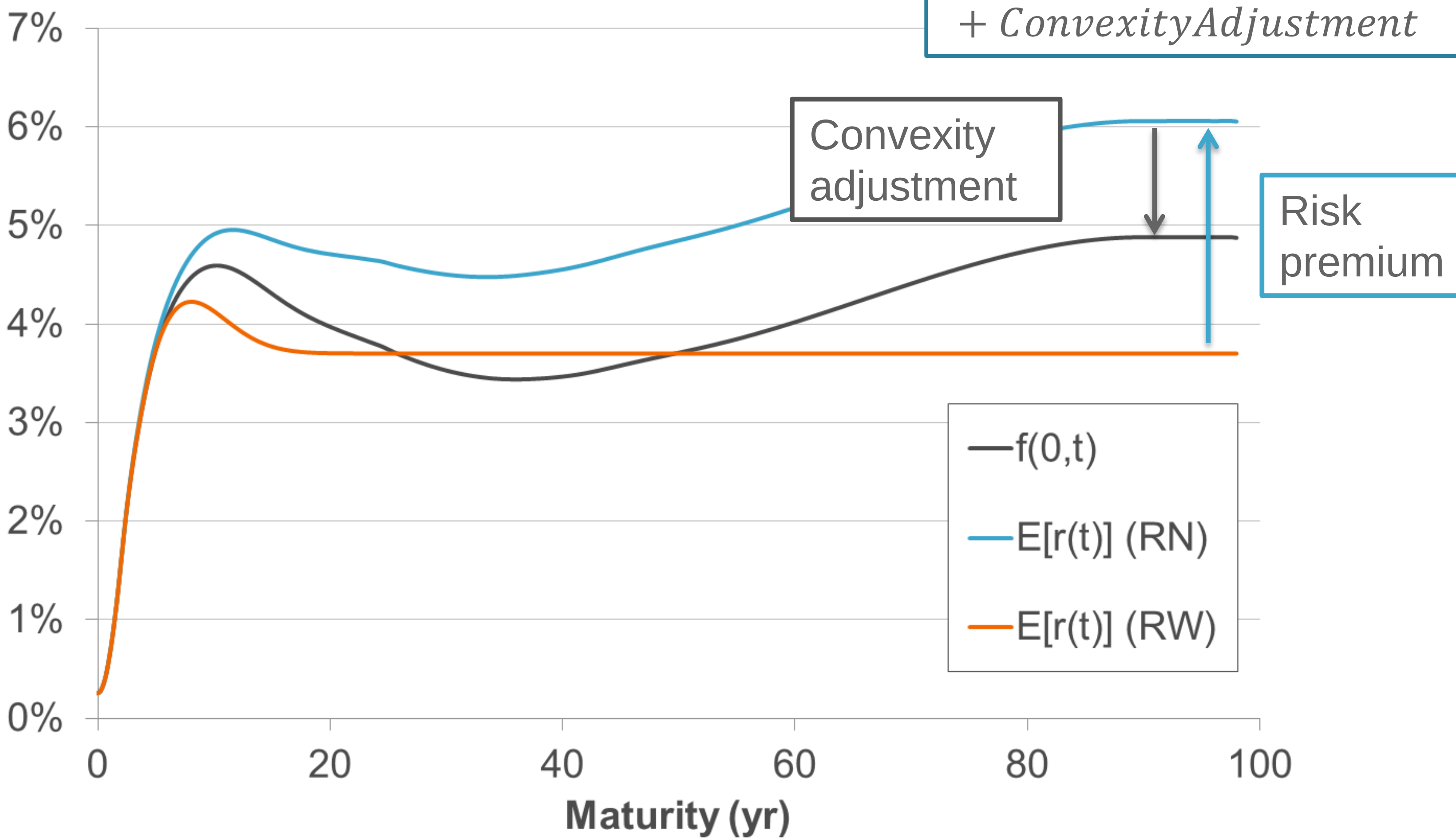


Real world modelling

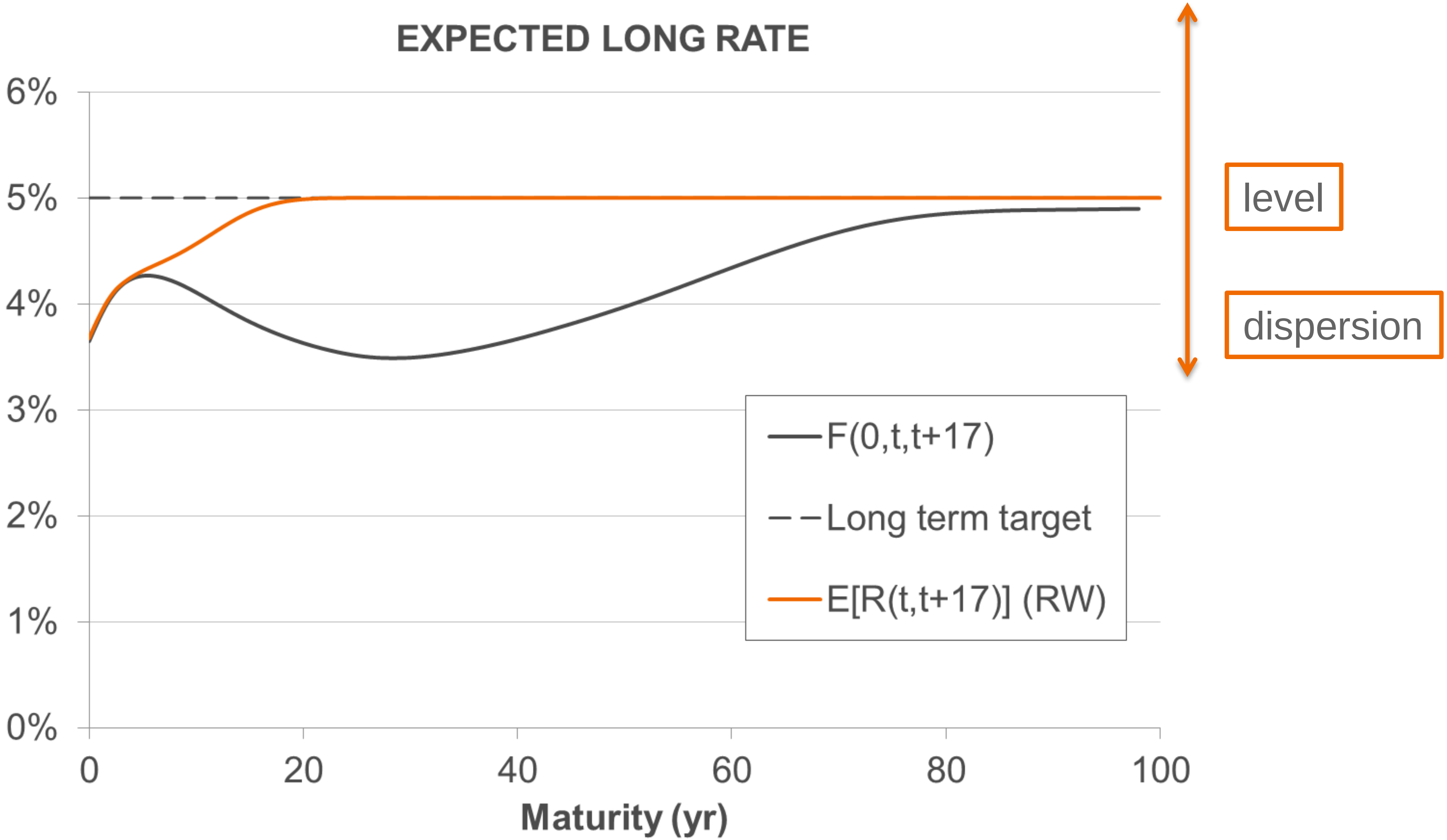
- Each path should look realistic. Paths should be equally likely.
- Ability to reproduce initial yield curve
- Future expectations of key variables (short rates, long rates, instantaneous inflation) should be in line with our views
  - Type of distribution is enforced by the model
- Width of distributions at different maturities should reflect our views

Risk premium and convexity

$$f(0,T) = E_P[r(T)] + RiskPremium + ConvexityAdjustment$$



Specification of target paths



Realistic distributions

- Levels
  - Exponentially weighted average of historical time series
- Dispersion
  - Exponentially weighted average of standard deviation of historical time series

Alternative: target quadratic (co)variations

Summary

- We want to price instruments and generate realistic probability distributions
- Modelling involves many choices and assumptions
- Empirical facts
- Foreign currency analogy for real rates and inflation

A little disclaimer

We review the calibration methodology on a regular basis and may modify, adjust or improve the methodology

Thank you

Any questions?