Flight Paths- Solution

This is a low threshold high ceiling question. It does not require any prior knowledge. Most students will be able to draw a map of flight paths. The questions then encourage deeper thinking.

After drawing the maps, you may then invite students to compare them. Correct solutions will have 29 lines, although the maps may look very different.

Below you can find answers to the questions.

1) There are 30 cities on the map.

2) The minimum number of lines needed is 29.

3) There can never be any loops, if you have really drawn the minimum number of lines you need.
   WHY: I can get from every dot in a loop to any other one by going around the loop in either direction. Assume I did draw a loop. If I removed a line from the loop, the dots would all still be connected (as I could just go the other way around the loop), but the number of lines would have gone down by one. Therefore, I did not have the least number of lines to start with in my graph.
   NOTE: The above argument is an example of a mathematical proof[1]. In particular, here we have used a way of reasoning that in mathematical logic is called "proof by contradiction".

4) No, there is not only one way. The actual maps you can draw may be quite different, but the number of lines will always be 29.

5) If we add another city there will be 31 cities and the minimum number of lines we need is 30.

6) If there are 32 cities on the map, we will need 31 lines to connect them all. There is a general pattern: if we have a certain number of cities on the map (to save characters we denote the number of cities on the map by *n*, then we will need *(n-1)* lines to connect them all, i. e. the number of cities minus one.

[1] A proof in Mathematics is a solid logical argument, that no one would be able to dismiss. It is based on logical steps rather than empirical observation.

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WHY: Pick any starting dot. If we connect another dot to it by a line, our number of dots and lines goes up by one. So now we have two dots that are connected and one line. If we connect a third dot to any of the two dots, we have three dots that are connected and two lines. We keep going, every time we find a dot that is not yet connected to our flight map, and we join it to any of the points that are already connected together. Note that every time we connect a new dot to our map, we add a single line.
If we keep going this way, the number of lines we need by the end will be equal to the number of dots minus one.

Note that our method will connect all dots and use the minimum number of lines: if we remove any line from what we have, the dots will not be connected anymore.

This kind of reasoning resembles a mental process mathematicians often go through. They observe and analyse particular cases, then they try to spot a general pattern and make a guess. Then they try to construct a mathematical argument to prove that their guess was correct.

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