Setting the risk appetite in presence of systemic risk

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Motivation

- In times of crisis, most of the financial assets move together and become very correlated.
  Last example : the 2008/09 crisis, with the interconnectedness of financial markets (see e.g. the *Systemic Risk Survey of the bank of England* available on line).
  A small fact, like turmoils in the sub-prime in the US real-estate market, a relatively small market compared to the whole US real-estate market, can trigger a major financial crisis that extends to all markets and all regions in the globe.

- Every financial crisis reveals the importance of systemic risk which manifests itself by the appearance of dependence structures (that were not deemed important during normal times) and the breakdown of the diversification benefits, with, in consequence, a big impact on the risk appetite of investors.

- For any financial institution, it is important to be aware of the limits to diversification, while, for researchers in this field, studying the mechanisms that hamper diversification is crucial for the understanding of the dynamics of the system (see e.g. Caballero et al., Cont et al., and references therein).
  $\hookrightarrow$ Both risk management and research on risk would enhance our capacity to survive the inevitable failures of diversification.

- Aim of this study : to understand and point out the boundaries to risk appetite with the right choice of risk measure for settings the limits under the presence of systemic risk.

- Which way ? via a simple stochastic modelling : combining two generating stochastic processes that, through their mixture, produces in the resulting process a non-diversifiable component, identified to a systemic risk

- Does the choice of the risk measure matter ? yes, it does.

Outline

1. Systemic vs systematic risk
2. Risk appetite, diversification and limit setting
3. An approach to systemic risk via a simple stochastic modelling
4. The influence of the choice of risk measure on risk appetite
5. Conclusion

Systemic vs systematic risk

- In the Report to G20 Finance Ministers and Governors by the Financial Stability Board, *Guidance to Assess the Systemic Importance of Financial Institutions, Markets and Instruments : Initial Considerations* (April 2009), systemic risk defined as

  'a risk of disruption to financial services that is caused by an impairment of all or parts of the financial system and that has the potential to cause serious negative consequences for the real economy.'
- In the economic literature, distinction between :
  - ▷ systemic risk : contagion effects
  - ▷ systematic risk : existence of an element that cannot be diversified away
- In mathematics, no clear accepted math. def. of either phenomenon
- So systemic risk manifests itself by a breakdown of the benefit of diversification, due either to contagion or to the presence of a non-diversifiable risk.

↪ We propose a mechanism equivalent to contagion as it happens simultaneously on many risks (contagion), but could also be assimilated to a non-diversifiable element coming from the mixture of two distributions. However, this model does not contain a time component that the term contagion would imply.

Risk appetite and diversification benefit

▷ Risk appetite

- Risk appetite : type of risk the company wants to take, e.g. P&C (long or short tail), Life insurance, which countries ?
- Risk appetite cannot be infinite ⇒ to define a tolerance to the risk, which means to define limits
- Generally, limits are given in terms of exposure to the risk : premiums, number of contracts, maximum limit of contracts...

  ↪ But it can be very approximative, as e.g. premium can vary for the same risk, depending on the market
- More subtle and adequate to give the limits in terms of risks, using a modelling for risk

  ↪ It allows to measure the risk, and the limits to risk appetite can then be defined in terms of capital allocated to the risk

- This allocated risk capital depends on the risk itself, but also on the diversification benefits associated to a risk portfolio
- It requires to measure the risk itself and the diversification benefits. For this, we need to consider an adequate risk measure to determine the capital
- We are going to use 2 standard risk measures, VaR and TVaR, and compare their efficiency to set the limits

▷ Framework

Suppose an insurance company has underwritten N policies of a given risk. To price these policies, the company must know the underlying probability distribution of this risk.

Assuming a portfolio of N similar policies, the technical risk premium for 1 policy in the portfolio is given by

$$P = (1+a)\,\mathbb{E}[L^{(1)}] + R, \quad 0 < a < 1$$

where $L^{(1)}$= loss incurred by 1 policy (of the portfolio),
$e = a\mathbb{E}[L^{(1)}]$= expenses incurred by the insurer

and R= risk loading per policy given by

$$R = \eta\left(\frac{\rho(L^{(N)})}{N} - \mathbb{E}[L^{(1)}]\right)$$

where η=cost of capital, ρ= risk measure, $L^{(N)}$= total loss of the portfolio.

▷ Diversification benefit

Diversification performance for a portfolio of N risks : measured via the diversification benefit $D_{n,\alpha}$ at a threshold α $(0 < \alpha < 1)$ defined by

$$D_{n,\alpha} = 1 - \frac{\rho_\alpha(L^{(N)})}{\sum_{i=1}^{n} \rho_\alpha(L_i)} = 1 - \frac{R^{(n)} + \eta\mathbb{E}[L^{(1)}]}{\frac{1}{n}\sum_{i=1}^{n} R^{(i)} + \eta\mathbb{E}[L^{(1)}]}$$

This indicator helps to determine the optimal portfolio of the company since diversification reduces the risk and thus enhances the performance.

Remark : $D_{n,\alpha}$ is not a universal indicator as it depends on the number of the risks undertaken and on the chosen risk measure ρ.

▷ Risk measures

- Value-at-Risk (VaR)

The Value-at-Risk with a confidence level α is defined for a risk L by

$$\mathrm{VaR}_\alpha(L) = inf\{q \in \mathbb{R} : \mathbb{P}(L > q) \leq 1 - \alpha\} = inf\{q \in \mathbb{R} : F_L(q) \geq \alpha\}$$

where q is the level of loss that corresponds to a VaR_α (simply the quantile of L of order α), and F_L the cdf of L.

Main drawback of the VaR :

The subadditivity fails to hold for VaR **in general**. It contradicts our notion that there should be a diversification benefit associated with merging the portfolios.
Cq : a decentralization of risk management using VaR is difficult since we cannot be sure that by aggregating VaR numbers for different portfolios or business units we will obtain a bound for the overall risk of the enterprise.

Rk : VaR gives no information about the severity of losses which occur with a probability less than $1 - \alpha$.

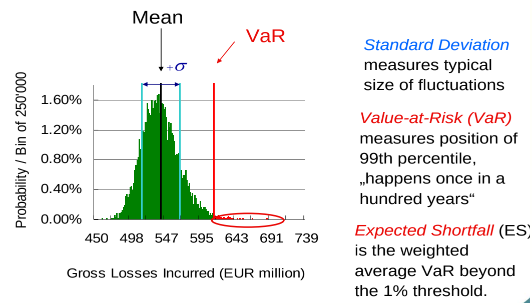
- Tail Value-at-Risk (TVaR) or Expected Shortfall (ES)

  The tail Value-at-Risk or Expected Shortfall at a confidence level α of L satisfies

$$\mathrm{TVaR}_\alpha(L) = ES_\alpha(L) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u(L)\mathrm{d}u \underset{F_L \text{ contin.}}{=} \mathbb{E}[L \mid L > VaR_\alpha(L)]$$

  which may be seen as the average over all losses larger than VaR_α
  Rk : ES is a coherent risk measure (will replace VaR in Basel 3)

- To summarize (see also Emmer et al. for a discussion on the use of risk measures in practice)



Stochastic modeling of systemic risk

- A toy model

  To have a better understanding of how insurances price risks and what problems they are facing, we present here a simple example, where the risk is well quantifiable.

  We assume an insurance customer approaches a company with the aim to insure a risk modeled by the throwing of a die (measurable uncertainty) as follows :

  - the customer must pay 10 EUR every time a die displays a 6 and nothing otherwise
  - he throws the die 6 times

  The customer would like to know the price an insurance company is going to ask him to cover such risk. It means we start by calculating the risk premium for one customer (i.e. one insurance policy).

  Consider a fair game, when the die is unbiased and it is equally probable to get one of the six faces of the die.

Let X be a Bernoulli random variable (rv) defined on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ representing the loss obtained when throwing the unbiased die, i.e when obtaining a "6".

$$X = \begin{cases} 1 & \text{with probability} \quad p = 1/6 \\ 0 & \text{with probability} \quad 1 - p \end{cases}$$

The loss amount when playing once is modeled by lX, with $l = 10$ EUR (in our example).
Let $(X_i, i = 1, \ldots, n)$ be a n-sample with parent rv X, corresponding to the sequence of throws when playing n times (independent games). In our example $n = 6$. The number of losses after n games is modeled by

$$S_n = \sum_{i=1}^{n} X_i \sim \mathcal{B}(n, p)$$

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TABLE: The loss distribution for a fair die with $n = 6$ throws.

# of "6" k	Loss per game $l\,X(\omega)$	Probab. Mass $\mathbb{P}[S_n = k]$	Cdf $\mathbb{P}[S_n \le k]$
0	0	33.490%	33.490%
1	10	40.188%	73.678%
2	20	20.094%	93.771%
3	30	5.358%	99.130% $\simeq VaR_{99}$
4	40	0.804%	99.934%
5	50	0.064%	99.998%
6	60	0.002%	100.000%

We generalize the previous approach for several customers, which means considering a portfolio containing N identical and independent policies. The idea is that the more policies we have, the more diversified is the portfolio of policies, thus the risk and the cost of capital are diminished.
We will examine how the risk is reduced, when increasing the size N of the portfolio.

We take the same example, now with N policyholders, each one paying EUR 10 in each case a 6 appears on a die at 6 throws.
The total number of losses in the portfolio of N policies with n throws can then be represented by the new rv

$$S_{Nn} = \sum_{i=1}^{Nn} X_i \sim \mathcal{B}(Nn, p)$$

Here we fix a threshold of $\alpha = 99\%$ and assume a cost of capital of 15% before tax.
Recall that the Risk loading R per policy is defined by $R = \eta \left(\frac{\rho(L)}{N} - nl\mathbb{E}(X) \right)$.
Then, for instance for $\rho = VaR_{99}$ and 1 policy ($N = 1$), we obtain $R = 0.15 \times (30 - 10) = 3$ (since $VaR_{99} = 30$, $n\mathbb{E}(X) = 6 \times \frac{1}{6} = 1$ and $l = 10$)

TABLE: Influence of diversification on the premium : the Risk loading per policy as a function of the number of policies in the portfolio.

Number N of Policies	Risk Loading R with $\rho = VaR(\alpha)$	Risk Loading R with $\rho = TVaR(\alpha)$
1	3.000	3.226
5	1.500	1.644
10	1.050	1.164
50	0.450	0.510
100	0.330	0.372
1'000	0.102	0.116
10'000	0.032	0.037

- A more realistic model

Assume that each policy is exposed n times to this risk $\Rightarrow$ in a portfolio of N policies, the risk may occur $n \times N$ times.

For a realistic description of a crisis, we assume that at each of the n exposures to the risk X, in a state of systemic risk, the entire portfolio will be touched by the same increased probability of loss, whereas, in a normal state, the entire portfolio will be subject to the same equilibrium probability of loss.

For this modeling, we introduce $(\mathbf{X}_j,\ j=1,\ldots,n)$ where the vector $\mathbf{X}_j$ is defined by $\mathbf{X}_j = (X_{1j},\ldots,X_{Nj})^T$. Hence the total loss amount S_{Nn} can be rewritten as

$$S_{Nn} = \sum_{j=1}^{n} \tilde{S}^{(j)} \quad \text{where} \quad \tilde{S}^{(j)} = \sum_{i=1}^{N} X_{ij}$$ (sum over all policies for a given risk)

To describe the state of crisis, we introduce a rv U that can take two possible values denoted by 1 (crisis) and 0 (normal) ; U can then be modeled by a Bernoulli $\mathcal{B}(\tilde{p})$, $0 < \tilde{p} << 1$.
U will be identified to the occurrence of a state of systemic risk.
If $\tilde{p}$ could mathematically take any value between 0 and 1, we choose it here to be very small since we want to explore rare events.
Under our assumption that at each of the n exposures to the risk, if a crisis, the entire portfolio will be hit, it means to define a normal $(U = 0)$ or a crisis $(U = 1)$ state on each vector $\mathbf{X}_j$, $1 \leq j \leq n$; it comes back to define a sequence of iid rv's $(U_j, j = 1, \ldots, n)$ with parent rv U.
Suppose that the dependence between the risks is totally captured by U.
We assume that $\tilde{S}^{(j)}$ follows a Binomial distribution whose probability depends on U_j :

$$\tilde{S}^{(j)} \mid (U_j = 1) \sim \mathcal{B}(N, q) \quad \text{and} \quad \tilde{S}^{(j)} \mid (U_j = 0) \sim \mathcal{B}(N, p)$$

with $q >> p$.
Note that these conditional rv's are independent (but unconditionally dependent).

Let us introduce the event A_l defined, for $l = 0, \ldots, n$, as

$$A_l := \{l \text{ vectors } \mathbf{X}_j \text{ are exposed to a crisis state and } n-l \text{ to a normal state}\}$$
$$= \left(\sum_{j=1}^{n} U_j = l\right)$$

whose probability is given by

$$\mathbb{P}(A_l) = \mathbb{P}\left(\sum_{j=1}^{n} U_j = l\right) = \binom{n}{l} \tilde{p}^l \,(1-\tilde{p})^{n-l}.$$

We can then write that

$$\mathbb{P}(S_{Nn} = k) = \sum_{l=0}^{n} \mathbb{P}(S_{Nn} = k \mid A_l)\mathbb{P}(A_l) = \sum_{l=0}^{n} \binom{n}{l} \tilde{p}^l \,(1-\tilde{p})^{n-l}\, \mathbb{P}\left[\tilde{S}_q^{(l)} + \tilde{S}_p^{(n-l)} = k\right]$$

with, by conditional independence,

$$\tilde{S}_q^{(l)} = \sum_{j=1}^{l} \left(\tilde{S}^{(j)} \mid U_j = 1\right) \sim \mathcal{B}(Nl, q)\,; \quad \tilde{S}_p^{(n-l)} = \sum_{j=1}^{n-l} \left(\tilde{S}^{(j)} \mid U_j = 0\right) \sim \mathcal{B}(N(n-l), p)$$

Numerical example.

We choose the number of times 1 policy is exposed to the risk, as $n = 6$. Unit loss l fixed to $l = 10$, probability of occurrence of a risk given a crisis = $q = 0.5$, proba given a normal state = $p = 1/6 = 0.17$.

Probability of occurrence of a systemic risk = $\tilde{p}$; will be chosen very small, from 0.1% to 10%

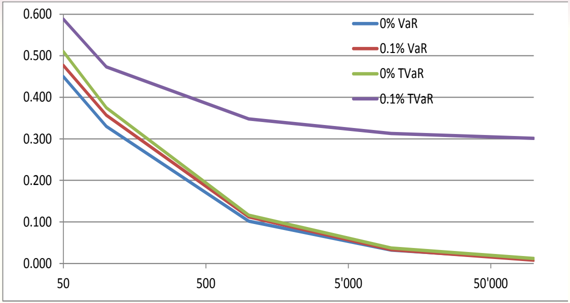
In this case, even if relatively simple, we cannot directly use an explicit expression for the distributions. We have to go through Monte-Carlo simulations.

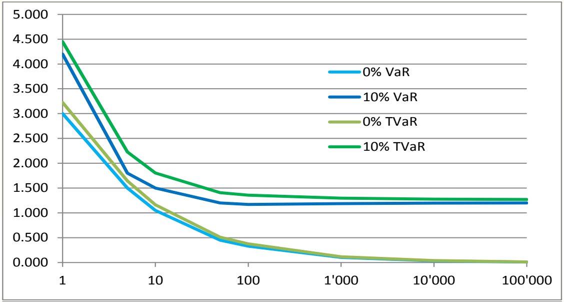
At each of the n exposures to the risk, we first have to choose between a normal or a crisis state.

To get enough of the crisis states, we need to do enough simulations, and then average over all the simulations. The results shown in the Table below are obtained with 10 million simulations. (We ran it also with 1 and 20 million simulations to check the convergence.)

TABLE: The Risk loading per policy as a function of the probability of occurrence of a systemic risk in the portfolio using VaR and TVaR measures with $\alpha = 99\%$. The probability of giving a loss in a state of systemic risk is chosen to be $q = 50\%$.

Risk measure ρ	Number N of Policies	Risk Loading R in a normal state $\tilde{p} = 0$	Risk Loading R with occurrence of a crisis state $\tilde{p} = 0.1\%$	$\tilde{p} = 1.0\%$	$\tilde{p} = 5.0\%$	$\tilde{p} = 10.0\%$
VaR						
	1	3.000	2.997	2.969	4.350	4.200
	5	1.500	1.497	1.470	1.650	1.800
	10	1.050	1.047	1.170	1.350	1.500
	50	0.450	0.477	0.690	0.990	1.200
	100	0.330	0.357	0.615	0.945	1.170
	1'000	0.102	0.112	0.517	0.882	1.186
	10'000	0.032	0.033	0.485	0.860	1.196
	100'000	0.010	0.008	0.475	0.853	1.199
TVaR						
	1	3.226	3.232	4.485	4.515	4.448
	5	1.644	1.792	1.870	2.056	2.226
	10	1.164	1.252	1.342	1.604	1.804
	50	0.510	0.588	0.824	1.183	1.408
	100	0.375	0.473	0.740	1.118	1.358
	1'000	0.116	0.348	0.605	1.013	1.295
	10'000	0.037	0.313	0.563	0.981	1.276
	100'000	0.012	0.301	0.550	0.970	1.269
$\mathbb{E}[L]/N$		*10.00*	*10.02*	*10.20*	*11.00*	*12.00*


0.600
0.500
0.400
0.300
0.200
0.100
0.000
50
500
5'000
50'000
0% VaR
0.1% VaR
0% TVaR
0.1% TVaR


5.000
4.500
4.000
3.500
3.000
2.500
2.000
1.500
1.000
0.500
0.000
1
10
100
1'000
10'000
100'000
0% VaR
10% VaR
0% TVaR
10% TVaR

We experience a part which is not diversifiable.

We also computed the case with 100'000 policies (since via Monte Carlo simulations). As expected, the risk loading in the normal state continues to decrease. In this state, it decreases by $\sqrt{10}$.

However, except for $\tilde{p} = 0.1\%$ in the VaR case, the decrease becomes very slow when we allow for a crisis state to occur.

For a high probability of occurrence of a crisis (1 every 10 years), the limit with VaR is reached already at 100 policies, while, with TVaR, it continues to slowly decrease.

Concerning the choice of risk measure, we observe for the case $N = 10'000$ and $\tilde{p} = 0.1\%$ that VaR is unable to catch the possible occurrence of a crisis state, which shows its limitation as a risk measure. Although we know that there is a part of the risk that is non-diversifiable, VaR does not catch it really when $N = 10'000$ or $100'000$ while TVaR does not decrease significantly between $10'000$ and $100'000$ reflecting the fact that the risk cannot be completely diversified away.

Comparing of the expectation and the variance of the total loss amount per policy, computed for the toy model and the dependent one :

Model	$\mathbb{E}[L]/N$	$var(L)/N^2$
toy (iid)	$ln\, p$	$\frac{l^2 n}{N}\, p(1-p)$
dependent	$ln\left(\tilde{p}\, q + (1-\tilde{p})\, p\right)$	$\frac{l^2 n}{N}\left(q(1-q)\tilde{p} + p(1-p)(1-\tilde{p})\right) + \mathbf{l^2 n\, (q-p)^2 \tilde{p}(1-\tilde{p})}$

The influence of the risk measure choice on risk appetite

- In general, both risk measures reveal the presence of systemic risk, whenever it appears with a probability $\geq 1\%$
- The risk factor loading per policy is around 5% higher with the TVaR than with the VaR $\Rightarrow$ the risk tolerance will be smaller with the TVaR than with the VaR
- More precisely, under the presence of systemic risk with
  - a very low probability of occurrence (0.1%, *i.e.* comparable to the probability of occurrence of a financial crisis), the VaR does not capture it (the diversification still takes place, as in normal time), whereas the TVaR does it well
  - a probability that coincides with the one we encountered historically (1 to 5 % ; *i.e.* every 100 to 20 years), VaR catches it, even if not as much as TVaR
  - a high probability (10% ; every 10 years), the VaR stops decreasing for any portfolio with more than 100 policies, whereas the TVaR decreases slowly to a limit, the non-diversifiable part

Conclusion

- Risk appetite must be related to a risk measure in order to establish the risk tolerance
- The risk tolerance is enforced by setting limits under risk modelling
- Limits are better set using risk modelling than risk exposure
- Limits need to account for the presence of systemic or systematic risk (for instance in life insurance, with pandemic or mortality trend that would affect the entire portfolio and cannot be diversified away.
- In real life, insurers (or practitioners of financial institution) have to pay special attention to the effects that can weaken the diversification benefits.
- The risk measure needs to catch those extreme risks and the limits will depend on the chosen risk measure

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